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Author index—1977

Aiken, Lewis R., Jr., Mathematics as a creative language, Mar., 251–55
Ashlock, Robert B., and Sherry P. Hubbard, Using flowcharts with first graders, Jan., 23–29

Billstein, Rick, A fun way to introduce probability, Jan., 39–42
Bonner, James, and Alexander Tobin, Elementary Mathematics Resource Teacher Program, Apr., 329–32
Bright, George W., IDEAS, Jan., 43–50; Mar., 211–18; May, 387–95
Brooks, Mattie J., The one-meter dash, Apr., 327–28
Brown, Irene M., From the file, Apr., 272
Bruni, James V., and Helene J. Silverman, Let's do it! Jan., 4–12; Feb., 96–102; Mar., 172–80; Apr., 265–72; May, 354–65
———, From blocks and model making to ratio and proportion, Mar., 172–80
———, Making patterns with a square, Apr., 265–72
———, More indoor games to motivate computational skills, May, 354–65
———, Using classification to interpret consumer information, Jan., 4–12
———, Using rectangles and squares to develop fraction concepts, Feb., 96–102
Burton, Grace M., Skinner, Piaget, Maslow, and the teachers of mathematics—strange companions? Mar., 246–50

Catanzano, Robert, and Debra J. Welker, Even Peanuts can do it, Feb., 146–47
Cathcart, W. George, Metric measurement: important curricular considerations, Feb., 158–60
Clason, Robert G., 1866—When the United States accepted the metric system, Jan., 56–62
Coppel, Arlene Cohen, Blueprint for ratio and scale, Feb., 125–26
Cowan, Richard A., Pentominoes for fun and learning, Mar., 188–90

Dahlquist, Joyce, Playing store for real, Mar., 208–10
Denman, Theresa I., editor, Etcetera, Feb., 136–39; Mar., 224–27; Apr., 314–17; May, 401–4
Dolson, Gene, and John C. Peterson, Property games, Jan., 36–38
Duea, Joan, and Earl Ockenga, IDEAS, Feb., 127–33; Apr., 303–10
Dutton, Wilbur H., Arithmetic education in Tokyo primary schools, Feb., 148–53

Edwards, Ronald R., Discoveries in geometry by folding and cutting, Mar., 196–98
Ekman, Mary Jean, From the file, May, 385
Epstein, Susan L., Weighing ideas, Apr., 293–97

Fennema, Elizabeth, and Julia A. Sherman, Sexual stereotyping and mathematics learning, May, 369–72
Fey, James T., Report of Study Group D: minimal competencies in mathematics, May, 405–7
Firl, Donald H., Fractions, decimals, and their futures, Mar., 238–40
Forys, Jan, From the file, Apr., 286

Gawronski, Jane Donnelly, editor, What's going on . . . , Jan., 31–32; Feb., 119–20; Mar., 199–200; Apr., 273–74

Haag, Vincent H., and Burt A. Kaufman, New math or old math?—the wrong question, Apr., 287–92
Haggard, Gary, and Ann Koch Schonberger, The game of Tri, Apr., 318–20
Hall, Cynthia T., and Donald E. Hall, The odometer in the addition algorithm, Jan., 18–21
Hall, Donald E., and Cynthia T. Hall, The odometer in the addition algorithm, Jan., 18–21
Hirschi, L. Edwin, The "new math," Mar., 241–45
Hubbard, Sherry P., and Robert B. Ashlock, Using flowcharts with first graders, Jan., 23–29
Hyatt, Donald W., M & M's candy: a statistical approach, Jan., 34

Jeffers, Verne G., Another use for the pegboard, May, 366–67
Jeffrey, Neil J., GCF and LCM on a geoboard, Jan., 63–64
Jencks, Stanley M., and Donald M. Peck, Hot and cold cubes, Jan., 70–71
Johnson, Martin L., Generating patterns from transformations, Mar., 191–95

Kaufman, Burt A., and Vincent H. Haag, New math or old math?—the wrong question, Apr., 287–92
Kepner, Henry S., and Lilane Koehn, Sex roles in mathematics—a study of the status of sex stereotypes in elementary mathematics texts, May, 379–85
Kidder, F. Richard, Euclidean transformations: elementary school spaceometry, Mar., 201–7
Klein, Judith S., Build a better geoboard, Jan., 85–86

Koehn, Lilane, and Henry S. Kepner, Sex roles in mathematics—a study of the status of sex stereotypes in elementary mathematics texts, May, 379–85
Kuhnke, Helen Fuesy, Update on sex role stereotyping in elementary mathematics textbooks, May, 373–76

Lai, Theodore, Bingo and the law of equal ignorance, Jan., 83–84
Leutzinger, Larry P., and Dean J. Mohr, Magic in the mathematics classroom, Apr., 298–302
Liedtke, Werner, The young child as problem solver, Apr., 333–38
Lott, Johnny W., Behold! A magic square, Mar., 228–29
Luke, Hope, and Phil Trembley, A model for a remedial mathematics program, Feb., 140–44
Lum, Susan, Meeting the mathematical needs in the middle grades, Mar., 233–37
Lund, Charles H., Tricks of the trade with cards, Feb., 104–11

Meyer, Rochelle Wilson, Unexpected classroom episodes can teach mathematics, Mar., 230–32
Mohr, Dean J., and Larry P. Leutzinger, Magic in the mathematics classroom, Apr., 298–302
Moody, William B., Arithmetic: must it be mechanical monotony and individualized boredom? Jan., 65–69
Montefusco, Nicholas, and Dennis Robidoux, An easy way to change repeating decimals to fractions—Nick's method, Jan., 81–82
Morgenstern, Frances B., and Morris Pincus, *Sometimes* material leads to discovery, Jan., 79–80
Morris, Janet P., Investigating symmetry in the primary grades, Mar., 181–86

Nicholas, Susan, Number concept for school beginners. Is it possible? Apr., 275–82

Ockenga, Earl, and Joan Duea, IDEAS, Feb., 127–33; Apr., 303–10

Peck, Donald M., and Stanley M. Jencks, Hot and cold cubes, Jan., 70–71
Pereira-Mendoza, Lionel, Graphing and prediction in the elementary school, Feb., 112–13
Peterson, John C., and Gene Dolson, Property games, Jan., 36–38
Prest, Gary, From the file, May, 407
Pincus, Morris, and Frances B. Morgenstern, *Sometimes* material leads to discovery, Jan., 79–80
Poage, Melvin, and Esther G. Poage, Is a picture worth a thousand words? May, 408–14
Poage, Melvin, and Esther G. Poage, Is a picture worth a thousand words? May, 408–14

Rathmell, Edward C., editor, New books for pupils, Jan., 51–52; Feb, 134; Mar., 220–21; Apr., 311–12; May, 397–98
Robidoux, Dennis, and Nicholas Montefusco, An easy way to change repeating decimals to fractions—Nick's method, Jan., 81–82
Robinson, Edith, editor, New books for teachers, Jan., 52–53; Mar., 221–22; Apr., 312–13; May, 398–99

Schonberger, Ann Koch, and Gary Haggard, The game of Tri, Apr., 318–20
Sherman, Julia A., and Elizabeth Fennema, Sexual stereotyping and mathematics learning, May, 369–72
Shumway, Richard J., editor, Metric materials, Jan., 54; Feb., 135–36; Mar., 223–24; Apr., 313–14; May, 400–401
Silverman, Helene J., and James V. Bruni, Let's do it! Jan., 4–12; Feb., 96–102; Mar., 172–80; Apr., 265–72; May, 354–65
———, From blocks and model making to ratio and proportion, Mar., 172–80
———, Making patterns with a square, Apr., 265–72
———, More indoor games to motivate computational skills, May, 354–65
———, Using classification to interpret consumer information, Jan., 4–12
———, Using rectangles and squares to develop fraction concepts, Feb., 96–102
Spitler, Gail, Multiplying by eleven—a place-value exploration, Feb., 122–24

Thornton, Carol A., A glance at the power of patterns, Feb., 154–57
Tobin, Alexander, and James Bonner, Elementary Mathematics Resource Teacher Program, Apr., 329–32
Tredway, Dan, Out of balance, Jan., 14–16
Trembley, Phil, and Hope Luke, A model for a remedial mathematics program, Feb., 140–44

Underhill, Robert G., Individualized teacher preparation for students *and* faculty through a CBTE model, Jan., 72–75

Verbeke, Linda M., An approach to teaching numeration systems, Jan., 76–78
Vrbka, Maxine M., Four television games for your classroom, Apr., 322–26

Webb, Leland F., Measuring, science, and history, Feb., 115–16
Wenner, William J., Compound subtraction—an easier way, Jan., 33–34
Welker, Debra J., and Robert Catanzano, Even Peanuts can do it, Feb., 146–47
Williams, Gail Atneosen, The Pythagorean theorem: a useful geometric tool for approximating $\sqrt{2}$, Apr., 284–86
Woodward, Ernest, Geometry with a Mira, Feb., 117–18

Subject index—1977

Accountability. *See* **Evaluation**

Addition and subtraction

Another use for the pegboard, 366–67
Compound subtraction—an easier way, 33–34
Hot and cold cubes, 70–71
IDEAS, 211–18
The odometer in the addition algorithm, 18–21

Affiliated groups. *See* **NCTM**

Aims. *See* **Objectives**

Algebra. *See* **Sentences**

Algorithm. *See also* **Addition and subtraction; Computation; Division; Multiplication**

Applications
From the file, 286
Playing store for real, 208–10

Area. *See* **Measurement**

Audiovisual materials. *See also* **Manipulative materials**
Etcetera, 136–39, 224–27; 401–4
Reviewing and viewing, 51–54; 134–39, 220–27; 311–17; 397–404

Basic skills. *See* **Computation**

Bilingual education. *See* **Teaching methods**

Books and materials
Books and materials, 75; 147; 215; 292; 417
Reviewing and viewing, 51–54; 134–39; 220–27; 311–17; 397–404

Calculators and computers
Readers' Dialogue, 416–17
What's going on . . . , 199–200, 273–74

Computation. *See also* **Addition and subtraction; Multiplication**
From the file, 407
IDEAS, 387–95
More indoor games to motivate computational skills, 354–65
Multiplying by eleven—a place-value exploration, 122–24
Number concept for school beginners. Is it possible? 275–82

Computers. *See* **Calculators and computers**

Curriculum
Fractions, decimals, and their futures, 238–40
Metric measurement: important curricular considerations, 158–60
A model for a remedial mathematics program, 140–44
The "new math," 241–45
New math or old math?—the wrong question, 287–92
What's going on . . . , 31–32; 199–200; 273–74; 377–78

Decimals. *See* **Fractions**

Diagnosis and remediation
What's going on . . . , 31–32; 273–74; 377–78

Division
A glance at the power of patterns, 154–57

Editorial comments. *See also* **Letters to the editor**
By way of introduction, 264
Thanks from the Editorial Panel, 161–62; 415

Equations. *See* **Sentences**

Equipment, mathematical. *See* **Manipulative materials**

Evaluation
What's going on . . . , 273–74

Experiment. *See* **Research**

Films and filmstrips. *See* **Audiovisual materials**

Fractions; fractional numbers
An easy way to change repeating decimals to fractions—Nick's method, 81–82
Fractions, decimals, and their futures, 238–40
From the file, 385
IDEAS, 43–50
Using rectangles and squares to develop fractions, 96–102
What's going on . . . , 119–20; 377–78

Games. *See also* **Recreational mathematics**
Four television games for your classroom, 322–26
The game of Tri, 318–20
More indoor games to motivate computational skills, 354–65
The one-meter dash, 327–28
Property games, 36–38
Tricks of the trade with cards, 104–11

Geometry
Discoveries in geometry by folding and cutting, 196–98
Euclidean transformations: elementary school spaceometry, 201–7
Generating patterns for transformations, 191–95
Geometry with a Mira, 117–18
IDEAS, 127–33
Making patterns with a square, 265–72
Pentominoes for fun and learning, 188–90
The Pythagorean theorem: a useful geometric tool for approximating $\sqrt{2}$, 284–86

Graphs and graphing
Graphing and prediction in the elementary school, 112–13

History of mathematics and mathematics education
1866—When the United States accepted the metric system, 56–62

Individual differences
What's going on . . . , 273–74

Individualized instruction
Meeting the mathematical needs in the middle grades, 233–37

Instructional materials. *See also* **Audiovisual materials; Books and materials**
Etcetera, 136–39; 314–17; 401–4
From the file, 272; 286; 385; 407
IDEAS, 43–50; 127–33, 211–18; 303–10; 387–95
Investigating symmetry in the primary grades, 181–86
Is a picture worth a thousand words? 408–14
Let's do it! 4–12; 96–102; 172–80; 265–72; 354–65
Metric materials, 224–27
New books for pupils, 51–52; 134; 220–21; 311–12; 397–98
Reviewing and viewing, 51–54; 134–39; 220–27; 311–17; 397–404
Sex roles in mathematics—a study of the status of sex stereotypes in elementary mathematics texts, 379–85
Tricks of the trade with cards, 104–11
Update on sex role stereotyping in elementary mathematics textbooks, 373–76
What's going on . . . , 31–32

Instructional television
What's going on . . . , 199–200

Instruments. *See* **Manipulative materials**

International. *See* **Mathematics in other countries**

Learning disabilities. *See* **Individual differences**

Learning process. *See* **Psychology; Teaching methods**

Letters to the editor
Readers' Dialogue, 45

Low achievers. *See* **Individual differences**

Manipulative materials
Another use for the pegboard, 366–67
Build a better geoboard, 85–86
Etcetera, 224–27; 314–17; 401–4
GCF and LCM on a geoboard, 63–64
Geometry with a Mira, 117–18
The odometer in the addition algorithm, 18–21
Out of balance, 14–16
Sometimes material leads to discovery, 79–80

Mathematics in other countries
Arithmetic education in Tokyo primary schools, 148–53
Report of Study Group D: minimal competencies in mathematics, 405–7
What's going on . . . , 377–78

Measurement
From blocks and model making to ratio and proportion, 172–80
Blueprint for ratio and scale, 125–26
Measuring, science, and history, 115–16
The one-meter dash, 327–28
Weighing ideas, 293–97
What's going on . . . , 31–32

Media. *See* **Audiovisual materials; Books and materials; Instructional materials; Instructional television**

Metric system, metrication. *See also* **Measurement**
1866—When the United States accepted the metric system, 56–62
Metric materials, 54; 135–36; 223–24; 313–14; 400
Metric measurement: important curricular considerations, 158–60; 400–401
What's going on . . . , 199–200; 377–78

Models. *See* **Manipulative materials**

Money
From the file, 286

Multiplication
A glance at the power of patterns, 154–57

NCTM, Affiliated groups
NCTM Affiliated Group Officers, 340–51

NCTM, Committee reports
Annual Financial Report, 88–89
Auditor's Report, 89–90
Board Action on 1976 Delegate Assembly Resolutions, 164–66
Nominees for the 1978 Election, 166

NCTM, Finances. *See* **Committee reports**

NCTM, Meetings
Your Professional Dates, 91–92; 167–68, 258–60; 351–52; 421–22

NCTM, Membership. *See* **Committee reports**

NCTM, Officers, committees, projects, and representatives
Committees and Representatives, 1976–77 Supplemental List, 90–91
The Editorial Panel of the *Mathematics Teacher*, 87
Financial Policies Committee, 339

NCTM Affiliated Group Officers, 340–51
Nominations and Elections Committee, 256
Publications Committee, 163
Your Council at Work, 418–19

Notation and terminology

An approach to teaching numeration systems, 76–78

Numbers; number systems; number theory

GCF and LCM on a geoboard, 63–64
The Pythagorean theorem: a useful geometric tool for approximating $\sqrt{2}$, 284–86
Readers' Dialogue, 45; 416–17

Number sentences. *See* **Sentences**

Patterns

Behold! A magic square, 228–29
Generating patterns from transformations, 191–95
A glance at the power of patterns, 154–57
Making patterns with a square, 265–72
Pentominoes for fun and learning, 188–90
Readers' Dialogue, 416–17

Pedagogy. *See* **Psychology; Teaching methods**

Percentage. *See* **Fractions**

Probability

Bingo and the law of equal ignorance, 83–84
A fun way to introduce probability, 39–42
Readers' Dialogue, 416–17

Problem solving. *See also* **Psychology; Teaching methods**

From the file, 272
IDEAS, 303–10
The young child as problem solver, 333–38

Psychology

Euclidean transformations: elementary school spaceometry, 201–7
Even Peanuts can do it, 146–47
The game of Tri, 318–20
Is a picture worth a thousand words? 408–14
Mathematics as a creative language, 251–55
A model for a remedial mathematics program, 140–44
New math or old math?—the wrong question, 287–92
Number concept for school beginners. Is it possible? 275–82
Sex roles in mathematics—a study of the status of sex stereotypes in elementary mathematics texts, 379–85
Sexual stereotyping and mathematics learning, 369–72
Skinner, Piaget, Maslow, and the teachers of mathematics—strange companions? 246–50
Update on sex role stereotyping in elementary mathematics textbooks, 373–76

Recreational mathematics. *See also* **Games**

Magic in the mathematics classroom, 298–302
Readers' Dialogue, 416–17

Remediation. *See* **Diagnosis and remediation**

Reviews. *See* **Books and materials; Audiovisual materials; Instructional materials**

Science. *See* **Applications**

Self-paced instruction. *See* **Individualized instruction**

Sentences

Out of balance, 14–16

Sets

Property games, 36–38
Using classification to interpret consumer information, 4–12

Sexism. *See* **Psychology**

Statistics

M & M's candy: a statistical approach, 34

Subtraction. *See* **Addition and subtraction**

Symmetry. *See also* **Geometry**

Investigating symmetry in the primary grades, 181–86

Teacher preparation

New books for teachers, 52–53; 221–22; 312–13; 398–99
What's going on . . . , 119–20

Teaching methods. *See also* **Ability grouping; Behavioral objectives; Computer-assisted instruction; Diagnosis and remediation; Discovery; Individualized instruction; Laboratory methods; Problem solving; Psychology**

Arithmetic: must it be mechanical monotony and individualized boredom?, 65–69
Elementary Mathematics Resource Teacher Program, 329–32
From blocks and model making to ratio and proportion, 172–80
Individualized teacher preparation for students *and* faculty through a CBTE model, 72–75
Let's do it! 4–12; 96–102; 172–80; 265–72; 354–65
A model for a remedial mathematics program, 140–44
New math or old math?—the wrong question, 287–92

Number concept for school beginners. Is it possible? 275–82
Skinner, Piaget, Maslow, and the teachers of mathematics—strange companions, 246–50
Sometimes material leads to discovery, 79–80
Unexpected classroom episodes can teach mathematics, 230–32
Using flowcharts with first graders, 23–29
Weighing ideas, 293–97
What's going on . . . , 31–32; 119–20; 199–200

Textbooks. *See* **Books and materials**

Time. *See* **Measurement**

Visual aids. *See* **Audiovisual materials; Manipulative materials**

Vocabulary. *See* **Notation and terminology**

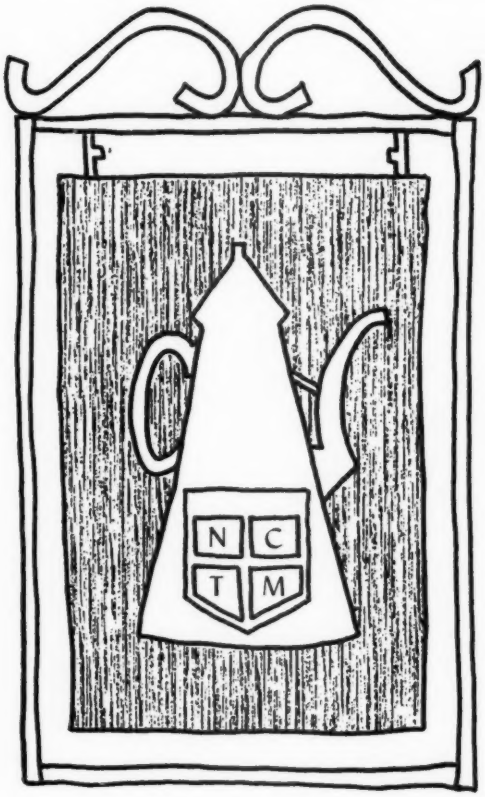
Volume. *See* **Measurement**

Worksheets. *See* **Instructional materials**

Correction

Librarians please note: A notice that the April 1977 issue of the *Arithmetic Teacher* was the last issue in Volume 24, the current volume, was carried by mistake in the April issue. Volume 24 ends in May 1977. It runs from January through May 1977. Volume 25 will begin in October 1977 and run through May 1978.

Winston-Salem Meeting



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November 3-5, 1977

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VOLUME 24, NUMBER 1

JANUARY 1977

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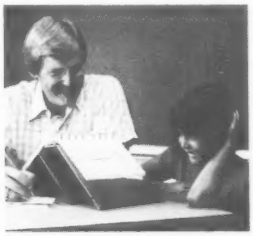

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The Arithmetic Teacher is devoted to the interests of elementary school teachers, students majoring in elementary education, and college and university professors responsible for teacher education programs in mathematics for elementary school teachers.

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The Arithmetic Teacher

JANUARY 1977 Vol. 24, No. 1

4 **Let's do it!** *James V. Bruni and Helene J. Silverman*
Using classification to interpret consumer information

14 Out of balance, *Dan Tredway*

18 The odometer in the addition algorithm, *Donald E. Hall and Cynthia T. Hall*

23 Using flowcharts with first graders, *Sherry P. Hubbard and Robert B. Ashlock*

31 **What's going on . . .**

33 Compound subtraction—an easier way, *William J. Wenner*

34 M & M's candy: a statistical approach, *Donald W. Hyatt*

36 Property games, *John C. Peterson and Gene Dolson*

39 A fun way to introduce probability, *Rick Billstein*

43 **IDEAS**, *prepared by George W. Bright*

45 Readers' Dialogue

51 **Reviewing and viewing**, *edited by Edward C. Rathmell, Edith Robinson, and Richard J. Shumway*

56 1866—When the United States accepted the metric system, *Robert G. Clason*

63 GCF and LCM on a geoboard, *Neil J. Jeffrey*

65 Arithmetic: must it be mechanical monotony and individualized boredom? *William B Moody*

70 Hot & cold cubes, *Stanley M. Jencks and Donald M. Peck*

72 Individualized teacher preparation programs for students *and* faculty through a CBTE model, *Robert G. Underhill*

75 Books and materials

76 An approach to teaching numeration systems, *Linda M. Verbeke*

79 *Sometimes* material leads to discovery, *Frances B. Morgenstern and Morris Pincus*

81 An easy way to change repeating decimals to fractions—Nick's method, *Dennis Robidoux and Nicholas Montefusco*

83 Bingo & the law of equal ignorance, *Theodore Lai*

85 Build a better geoboard. . . , *Judith S. Klein*

87 Your Council at Work
Editorial Panel of the *Mathematics Teacher*
Annual Financial Report
Auditor's Report
Committees and Representatives, 1976–77 Supplemental List

91 Your Professional Dates

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By **James V. Bruni** *and* **Helene J. Silverman,**
Herbert H. Lehman College, City University of New York

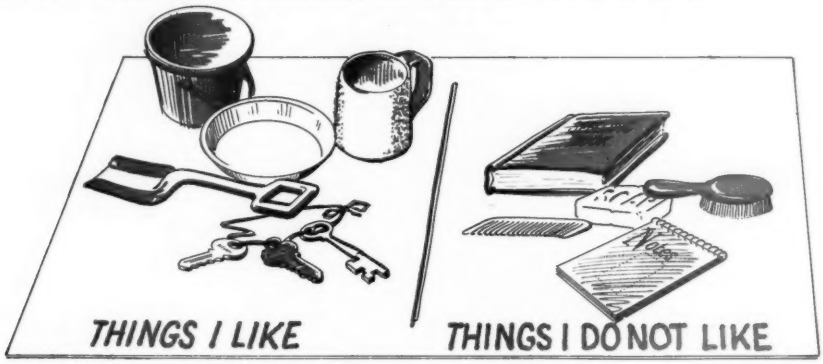
Using classification to interpret consumer information

In their daily lives children are showered with information presented in the form of schedules, tables, checklists, and charts that can be very puzzling unless the children have specific skills for interpreting the communications. Some materials come to children in no special order and may appear haphazard; the children then need to arrange the data in ways that show some pattern or format. Other materials may come arranged in some order, but children frequently do not know how to interpret the data that is provided.

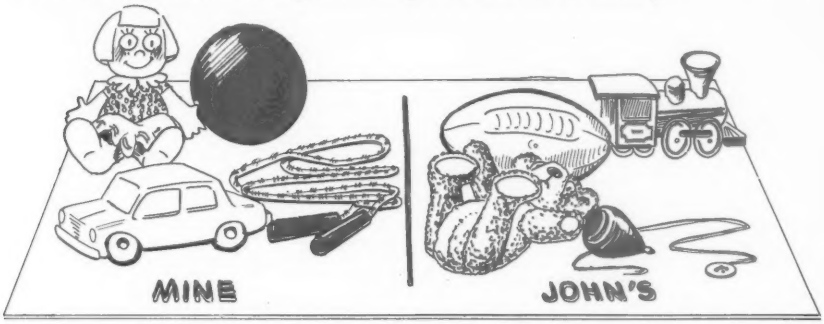
Many of the skills that are needed to interpret information are extensions of concepts commonly taught to children as they engage in experiences with attribute materials and sorting games. If we can help children to extend the skills of classification to arranging data obtained in other investigative activities, we can teach them to analyze and use the material presented to them as consumers.

SORTING

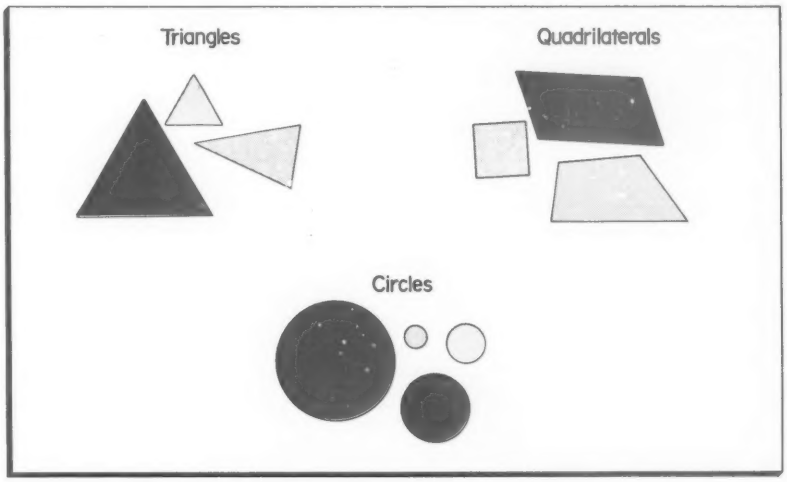
Most young children naturally arrange objects into sets. The arrangements may simply be objects that are interesting and objects that are to be ignored.

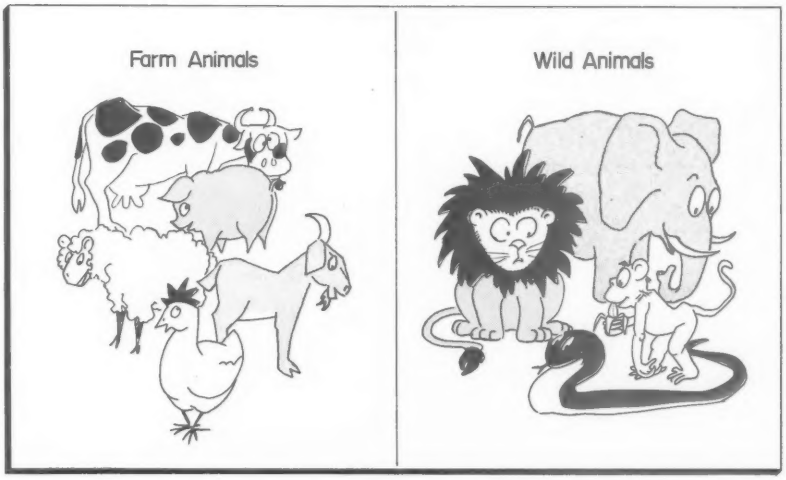


A more sophisticated arrangement may be based on ownership.



As children work with attribute games and materials they learn to classify objects on the basis of properties of the materials and to display the objects in sets.





Classroom information may be conveyed in a similar way.

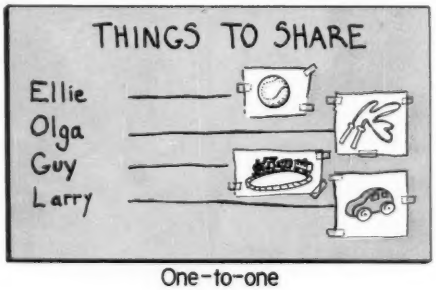
ART, MUSIC	MATHEMATICS, SCIENCE
Joey, Leba, Karin, Tony	Iris, Arthur, George, Bobby
LANGUAGE ARTS	**SOCIAL STUDIES**
Paula, Angel, Cora, Darlene	Ralph, Joe, Tamara, Ida

SPECIAL CLASSES

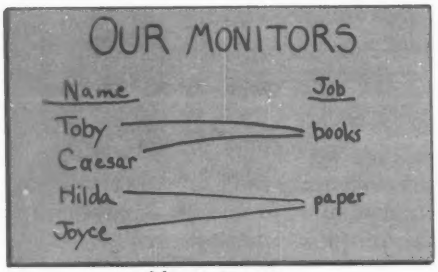
SPEECH	MUSIC
Robert	Regina
Tina	Michael
Amy	Angel
August	Carol
Paul	Donald

Matching

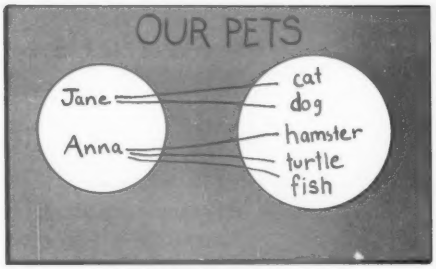
Children are also taught to match the elements of one set with those of another.



One-to-one



Many-to-one



One-to-many

THE CONSUMER WORLD

Applications of classification are at the heart of merchandising. How is a store set up? How is the merchandise grouped? Consumers regularly are faced with the task of buying things in a large supermarket or department store. How do they find what they want? One technique is to browse through all the departments looking for the needed merchandise. This approach is exactly what the business world hopes for; they assume that the consumer will be attracted to additional items on the way.

A more organized approach depends on the ability to use a list or chart and to understand the classification of merchandise into departments.

In the classroom, children may be asked to make up shopping lists from advertisements, or to bring real shopping lists from home.

Shopping List

facial tissues
milk
lettuce
tomatoes
bananas
oranges
napkins
swiss cheese
frozen orange juice
frozen cut beans
fish cakes
London broil
chicken wing
potatoes
yogurt

Tracy, Jan. 10

A listing of the typical classifications of merchandise in a supermarket is needed. This can be compiled by the class on a visit to a local market or through research in food merchandising journals. Items on the individual student's shopping list can then be classified according to the supermarket departments.

Classified Shopping List

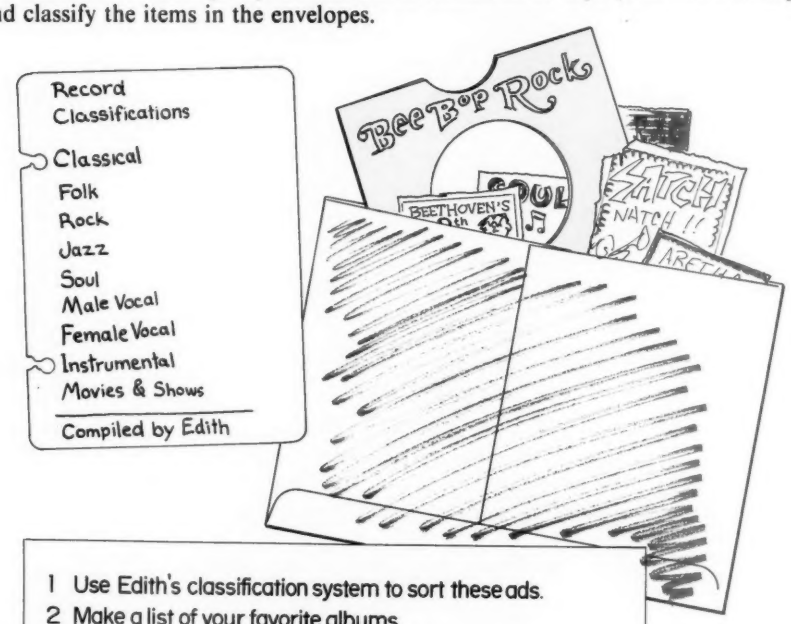
DAIRY	PRODUCE	PAPER GOODS
milk Swiss cheese yogurt	lettuce tomatoes potatoes bananas oranges	facial tissues napkins

FROZEN FOOD	FRESH FISH, MEAT, POULTRY
orange juice cut beans fish cakes	London broil chicken wings

BAKED GOODS

TRACY, JAN. 10

Shopping lists can also be developed for record stores, book stores, toy shops, and department stores. If mail-order catalogues, newspaper supplements, mailed flyers, and newspaper advertisements are used as resources, collections of advertisements can be put in envelopes. If classification systems for a variety of retail stores are available, a child or a group of children can select an evelope (or a set of evelopes) and classify the items in the envelopes.

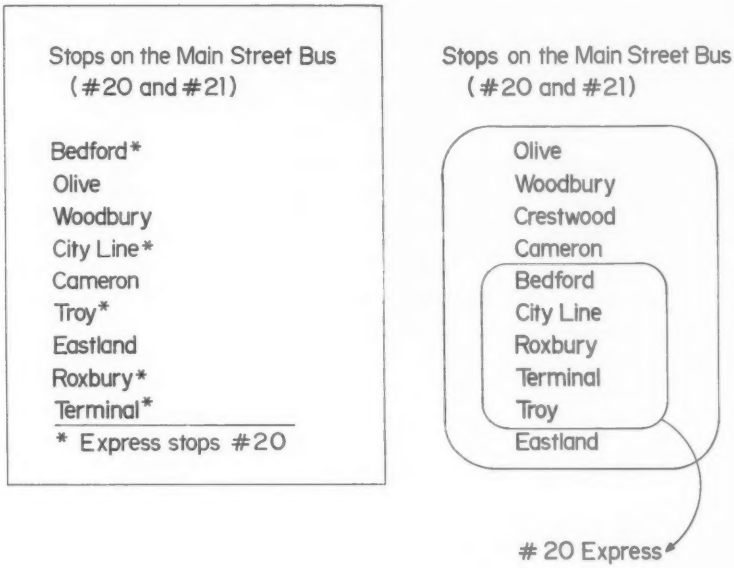


1 Use Edith's classification system to sort these ads.
2 Make a list of your favorite albums.
3 What kind of albums do you like ?
4 Might some albums be placed in more than one category ?

Extensions of the classification activity might include pricing and cost distribution of items, thus encouraging computation practice in a nonroutine manner.

Sets and subsets

Although we teach children to make subsets from a larger collection, we frequently neglect to show children how the idea of subsets is used in the consumer world. Children who have had experiences finding subsets of a group of attribute materials may need to be shown how to understand the information conveyed in a bus route schedule that superimposes the express bus stops on the local route.



Another method, which conserves the order of the stops on the bus route as well as indicates the local and express stops, can be developed.

Stops on the Main Street Bus (#20 and #21)

Street	Express	Local
Bedford	*	*
Olive		*
Woodbury		*
City Line	*	*
Crestwood		*
Cameron		*
Troy	*	*
Eastland		*
Roxbury	*	*
Terminal	*	*

The underlying concept in each case is that every express stop is also a local stop. For further practice, it may be useful to have a variety of train maps and bus maps available so that the children can practice reading the designations.

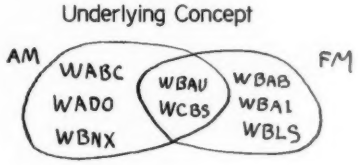
Children who have learned to classify sets that have elements in common can be taught to use this skill to help them to analyze other tables of data. It may be confusing, for example, to find two distinct listings for radio stations.

Radio Stations

Name	AM	FM
WABC	770	
WADO	1280	
WBAB		102.3
WBAI		99.5
WBAU	640	90.3*
WBLS		107.5
WBNX	1380	
WCBS	880	101.1*

*Received on AM and FM

Underlying Concept



In playing some of the attribute games, children often learn to arrange objects in rows and columns and to record data in tables.

	shape			
color	□	○	△	⬡
red	□	○	△	⬡
yellow	□	○	△	⬡
blue	□	○	△	⬡
green	□	○	△	⬡

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

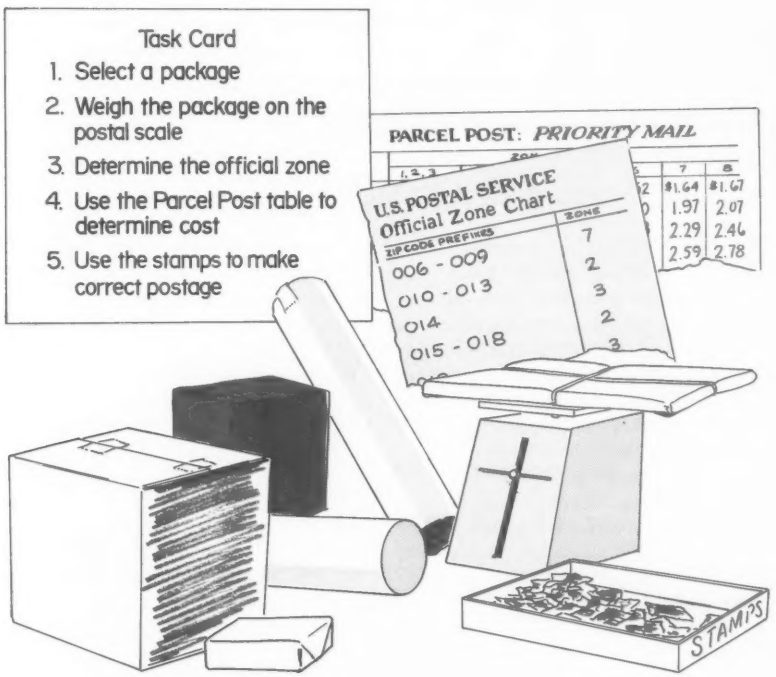
Much of our consumer information depends on the ability to interpret tables, to know what the numbers in the cells mean. Consider the parcel post table for priority mail. Two factors (attributes) need to be considered—weight and zone number. If

the zone number and the weight are known, the consumer locates the cell corresponding with the two entries.

PARCEL POST: *PRIORITY MAIL*

WEIGHT	ZONES					
	1, 2, 3	4	5	6	7	8
1.0 pound	$1.56	$ 1.58	$ 1.60	$ 1.62	$ 1.64	$ 1.67
1.5	1.73	1.77	1.84	1.90	1.97	2.07
2.0	1.89	1.96	2.07	2.18	2.29	2.46
2.5		15	2.29	2.43	2.59	2.78

Children can be given a list of zipcodes and a zone chart for their area. If a small mail scale is available, the children can actually weigh parcels in an activity center. If scales are not available, cards can be made up with an address and weight written on the card. To determine the postage for a package, the child or group of children would first use a zone chart to determine how many zones the piece must travel. Then, the weight is determined and the letter or parcel cost is indicated.



ASSISTING CHILDREN WITH THE LEARNING PROCESS

The daily papers, mails, catalogues, and magazines have a variety of consumer information. Once children are taught to work with a particular kind of table, they

can be encouraged to find other applications of the technique. In their own investigative work, they can be encouraged to present data in many ways.

In addition to experiences with making and interpreting charts and tables, it may be useful to teach children a process for analyzing the data that is ubiquitous in advertising. Children can be encouraged to ask such questions as the following:

1. How was the information collected?
2. How is the information organized?
3. What direct information can I get from this chart?
4. What additional inferences can be made from this data?
5. Can this data be extended?
6. How can this data be applied?

Data and the presentation of data may take many forms, but there are techniques that are common to the reading and analyzing of any data. These are what need to be stressed. Children also should realize that the same kind of information can be presented in a variety of ways. In any case, the main purpose of the instruction should be to foster understanding of the data that is being presented.

Reference

Bruni, James V., and Helene Silverman. "Making and Using Attribute Materials." *Arithmetic Teacher* 22 (February 1975):88–95.

ANNOUNCEMENT

The Comprehensive School Mathematics Program (CSMP) has completed the development of its K-4 curriculum and is presently creating instructional materials for grades 5-6 under a contract with the National Institute of Education.

CSMP is a curriculum development project of CEMREL, Inc., a nonprofit national educational laboratory. The CSMP elementary school program is for students of all ability levels, including the educable mentally retarded and the gifted. The program is based on a spiral approach to learning and a "pedagogy of situations." It makes extensive use of several nonverbal languages including the Papy Minicomputer, developed by Georges and Frederique Papy of the Belgian Center for the Pedagogy of Mathematics. (Frederique is presently the Associate Director for Research and Development for CSMP.)

During the 1976-1977 academic year approximately 27,000 youngsters in 40 school districts in 15 states are participating in a pilot test of the CSMP K-4 materials. Additional pilot sites are being sought for 1977-78. CSMP also has available a special secondary school program (grades 7-12) for gifted students.

For further information, write:

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Comprehensive School Mathematics Program
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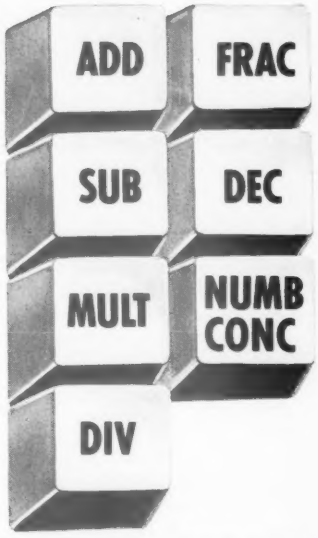
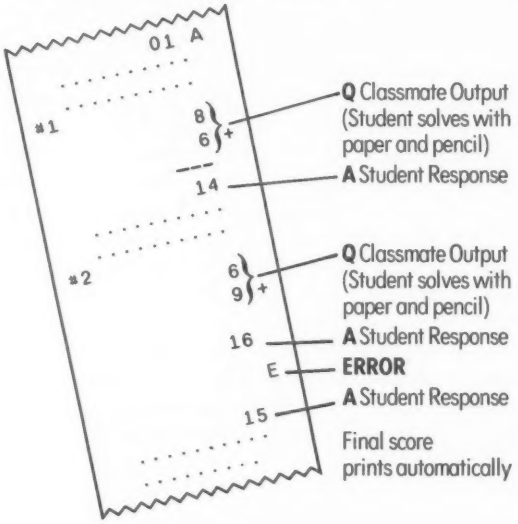


HERE ARE 7 OF THE 70 SEQUENTIAL SKILL PROGRAMS.

Program Number	Addition Skill
01	Basic Facts 1-9, Sums ≤ 18
02	2 digits + 2 digits, no regrouping
03	2 digits + 2 digits, regrouping in units
04	3 digits + 3 digits, regrouping in 10's
05	3 digits + 2 digits, regrouping in units and 10's
06	**ADDITION REVIEW**—2 addends—Constants and Limits
07	**ADDITION REVIEW**—3 addends—Constants and Limits

CLASSMATE 88 can generate an unlimited number of exercises for each skill program. The tape shows an example of the addition 01 program.



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Out of balance

DAN TREDWAY

A faculty member at Western State College of Colorado, Dan Tredway teaches undergraduate methods courses in elementary mathematics and inservice courses in the use of mathematics laboratories in elementary schools. He is currently involved with developing materials for individualized mathematics instruction.

The balance beam is an old, simple, easily constructed device that should be reexamined by most elementary teachers. This device, whether commercially manufactured or homemade, has been typically consigned to either the storage cabinet or kindergarten and first grade where its use is in assisting pupils in developing the concept of equalities and inequalities.

For example, the child may verify physically that $7 < 8$ by putting a washer at 7 on one side and 8 on the other side of the balance beam. (Fig. 1) The use of the balance beam may also be extended to the child's working with sums and addition. The child shows $4 + 3$ on one side and identifies the correct answer of 7 by showing it on the other side of the balance beam.

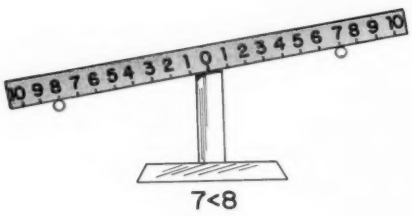


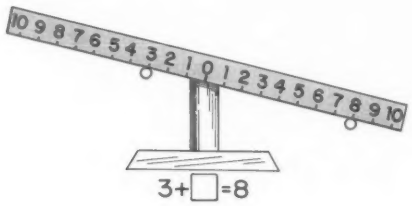
Fig. 1

The balance beam provides an easy tool for introduction to and review of all the basic processes. It is useful for the student working individually in a mathematics laboratory setting and for the teacher demonstrating an idea before a group. It can be used to both present and confirm correctness or incorrectness of results in any of the basic processes.

One of the situations in which the balance beam is very useful is in instruction where the missing addend is presented in a number sentence. Typically this is one of the most difficult concepts that seven-year-old children face. The arithmetic problem $3 + \square = 8$ gives the child for the first time an indicated operation symbol and a second operation to perform. In this situation the balance beam can be of use in assisting pupils to see that this number sentence, like others, is a statement of equality. It will allow a child to place the known quantities on opposite sides of the balance beam and to verify his hypotheses or establish the correct solution by trial and error. (Fig. 2)

In allowing the child to develop an understanding by this process you not only provide him with a solution; he also has instant reinforcement by balance or lack of balance on the beam. The pupil who is uncertain may check his written work in a physical manner and in a way that instantly identifies a correct or incorrect response. Also, at this level, the use of the balance beam provides a valuable experience with establishing addition as commutative, for the child can see that $6 + 5$

equals 5 + 6. The child also may be helped to regroup the 6 and 5 into a 10 and a 1 so that the solution to the expression will be written correctly.



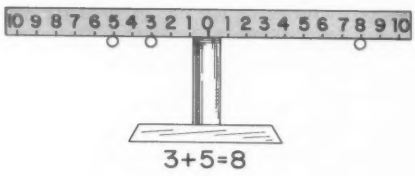


Fig. 2

In subtraction, the balance beam will assist the child in a variety of ways. A child may arrive at the solution of 8 − 5 = □ by placing 8 individual units in the ones place and then removing 5 of them and discovering what number on the other side will balance. At a slightly higher degree of sophistication, the same child may place an 8 on one side of the balance beam, a 5 on the other, and arrive at an appropriate balance of 3 as his solution.

A very interesting use of the balance beam is found in development of multiplication readiness and demonstration of the commutative property in multiplication. The child can express composite numbers in terms of the given number of one of the factors. For instance a child may express fifteen as three 5s or five 3s. He may also, in studying this, discover that five 3s equal three 5s through the use of the beam. He can express multiplication as regrouping by placing four 8s on the balance beam and then identifying the product as three 10s and 2. (Fig. 3) Children can easily identify basic multiplication facts and at the same time make a direct association between multiplication and repeated addition of equal factors.

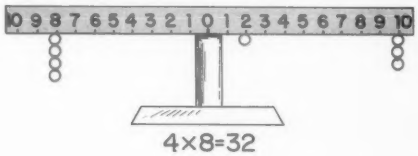


Fig. 3

While working with multiplication on the balance beam, basic division facts may be introduced. As an example, when placing a 24 on the balance beam with two 10s and a 4, the child may be asked to find the number of 6s that equals 24 and he can do this by physically putting washers on the balance beam until four 6s balance 24. (Fig. 4) By the same process he may also be introduced to dealing with remainders in division. After placing 19, a 10 and a 9, on one arm of the balance beam, the pupil may be directed to identify the number of groups of 3 that equal 19. When he arrives at 6 washers on the 3 peg, he will not have achieved the equality, and neither will he have equality when he places seven washers on the balance beam. However, if the child is given time to achieve a solution he will be able to identify that 6 groups of three and, by experimentation or knowledge, 1 more as 19. (Fig. 5)

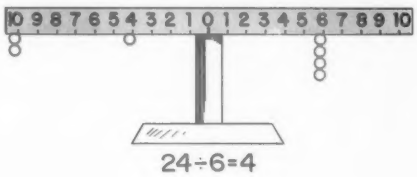


Fig. 4

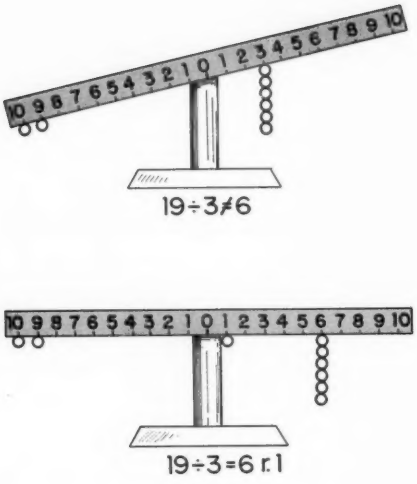


Fig. 5

In summary, the balance beam is a valuable tool for working with all of the basic processes. In a laboratory setting it is particularly valuable because it provides the pupil with an instant feedback of the correctness of his solution. The child is given a reinforcement on correct responses and a demonstration on equality or inequality at any given time during his computational experience. The use of the balance beam for multiplication and division and working with remainders is extremely important because it demonstrates the regrouping involved with physical materials and allows the child to make an association between operations.

In addition to being used in the above situations, the balance beam may very well be used with remedial children or on occasion as a device to allow pupils to demonstrate their knowledge of the operations in a review situation. It is an easy, almost indestructible device and one that certainly deserves more use than it receives at the present time.

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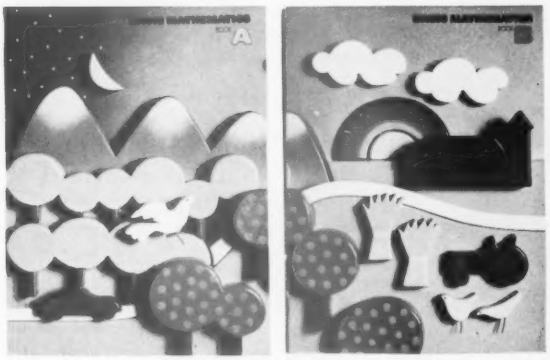

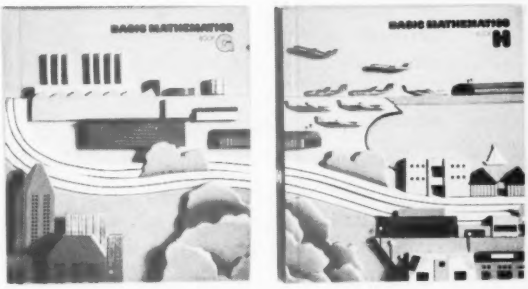

proper pacing
gives children time to learn by heart the basic number facts

gives children opportunity to discover by doing important ideas about the structure of our decimal number system

good instruction
develops number operations using pictures and diagrams in the mathematical language of numerals and operational signs, not in words

develops counting and measurement as ways of understanding the child's own physical world

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insures understanding of the mathematical concepts children must have for high school courses

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The odometer in the addition algorithm

DONALD E. HALL and
CYNTHIA T. HALL

Affiliated with the University of Massachusetts since 1960, Donald Hall has worked intensively in mathematics education, including presenting papers at both local and national NCTM conferences. His special area of mathematics research is quantitative judgment. Cynthia Hall has used the techniques described in this article in her work as a teacher at the Mark's Meadow Laboratory School, which is part of the university. She has taught in a variety of preservice and inservice positions and is coauthoring research in quantitative judgment.

In order for pupils to learn to "do" arithmetic the learning experience must be one in which the basic concept is grasped through meaningful, mathematically oriented activities. The learning environment also should include a methodology that stresses that—

- the *understandings* must precede the "doing,"
- the pupil's previous *discoveries* may lead him to invent many algorithms,
- a *variety* of approaches to the problem strengthens concepts,
- *readiness* for computational skills is a prerequisite for introduction to the skills,
- mastery comes through a *multitude* of experiences, rather than repeated drills.

The following activity is designed to strengthen the pupil's understanding of the addition algorithm with particular emphasis on the concepts of place value and renaming. It is based on the automobile odometer.

Oaktag provides a good material for construction. Any number of paired slits (from five to ten) provides place-value columns. Then strips of paper with the numerals 0 through 9 in column form can be inserted into the slits and the strips pasted to form circles. Headings placed above the columns indicate the ones, tens, hundreds, and so on. The odometer as pictured in figure 1 is then ready for operation.

The actual use of the odometer can be presented in a variety of ways, but its use should be preceded by experiences with abacuses. An imaginary trip may be a part of the introduction of the odometer. For example, "The automobile is ready for its journey. This car has never traveled on its own power. What will the odometer read?" The pupil should be able to turn his odometer to read the appropriate mileage. Then as the journey continues mile by mile (or kilometer by kilometer) the pupil will experience turning his own odometer from 00009 miles to 00010 miles, from 00099 miles to 00100 miles, and so on. There seem to be strategic points for understanding of place value and renaming in the Hindu-Arabic number system. With this device the pupil is guided to represent, for example, 10

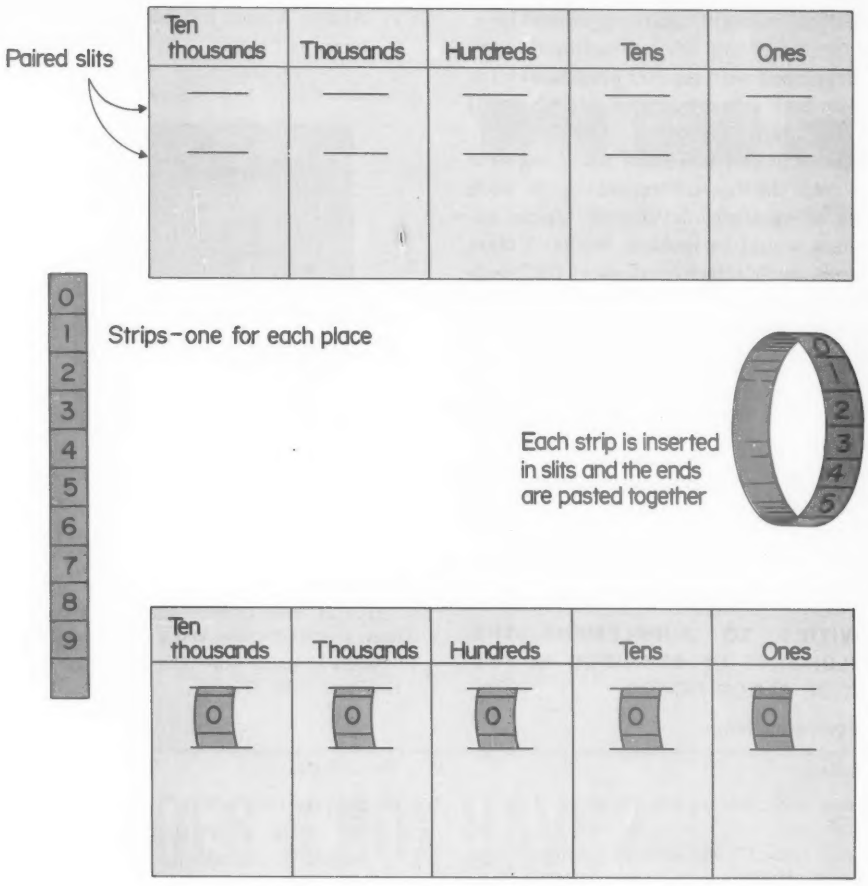

Fig. 1

ones as 1 ten and 0 ones, 10 tens as 1 hundred, 10 hundreds as 1 thousand, and so on.

Eventually, the pupil may reach a point where he initiates his own word problems, which can be solved by rotating the odometer mile by mile or using the addition algorithm. For example, John's car travels 16 miles the first day it is driven, and 4 miles the next. What will the odometer read? One pupil may turn the odometer from 0 to 16 miles and continue to show the mileage at 17, 18, 19, 20. Another pupil may reach a level where he notes that 16 + 4 = 20 is a shortcut for solving the problem without using the odometer. The scope of discussions, inventions, discoveries, and reviewing concepts with this one activity appears self-generating.

This activity has shown itself to be one in which the pupil enjoys constructing the device, and meaningful tangential inquiries occur as the pupil becomes involved in imagined or real journeys using the odometer to record the measurements. The pupil's understanding of place value, renaming, and the addition algorithm will be enriched although no one activity can provide the necessary mathematical experience for total concept development. For this reason additional activities that would supplement the development of renaming in the addi-

tion algorithm are also suggested here. These may be used for instructional study and developed with teacher guidance or for independent investigation with small groups of pupils.

These activities are but a small segment of a total lesson on renaming; a wide variety of carefully developed, logical experiences would be needed. Within a class of pupils, each activity will meet the needs of some pupils most of the time, but no one experience will provide learning for all pupils at a given age or grade level. The teacher's role is that of developing a flair for inventive, creative, different activities that are mathematically sound, actively inviting, and an exciting part of the pupil's learning. The activities can then be matched to the needs, strengths, and weaknesses of particular students.

ACTIVITIES TO SUPPLEMENT THE DEVELOPMENT OF RENAMING IN THE ADDITION ALGORITHM

1. Place-value bingo

Materials:

Sheets with nine squares, arranged 3 by 3. (Fig. 2)

Small cards (27) with numerals 1 through 9 and the words *hundreds*, *tens*, or *ones* on each.

Markers to cover squares.

Procedure:

a. Pupils number the nine squares on their sheets with the numerals 1 through 9 in any order.

b. Teacher or aide draws cards and reads "2 tens," "3 hundreds," "7 ones," and so on as the cards are drawn.

c. Players cover squares with markers when "their numbers" are called.

d. When three squares in a row, column, or a diagonal have been covered, the player raises his hand and markers are checked.

e. After a winner has been found, a new game is begun.

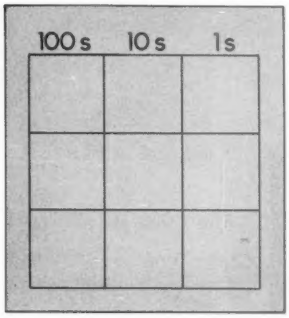

Fig. 2

2. Dice roll

Materials:

Dice of three different colors—one color for ones, another for tens, and a third for hundreds.

Sheets of paper with columns labeled 1s, 10s, 100s, and 1000s. (Fig. 3)

Procedure:

a. Players take turns. Each player rolls the dice and then records the numbers rolled in the appropriate columns on his paper.

b. On each turn after his first, the player records the new roll and adds this to his previous list. (Fig. 3)

c. First player to score a specified amount, say 3000, or the player with the highest score in five rolls wins. Players check each other's addition. Any error is subtracted from total score. Accuracy counts.

1000s	100s	10s	1s
	4	6	1
	2	3	1
	6	9	2

Fig. 3

3. Card trading

Materials:

For each 3 players, a set of 27 cards labeled as follows:

1 hundred,	10 tens,	100 ones
2 hundreds,	20 tens,	200 ones
3 hundreds,	30 tens,	300 ones
.	.	.
.	.	.
.	.	.
9 hundreds,	90 tens,	900 ones

Procedure:

a. Each player is dealt five cards. The rest of the cards are placed in a pack facedown in the center of the table. Play begins to the left of the dealer.

b. When it is a player's turn, any three matching cards are placed faceup on the table in front of the player. Example:

c. The player then asks the player to the left, "Do you have————?" (any one card needed).

d. If the answer is yes, the player takes the card. If the answer is no, the player draws one card from the pack.

e. The game ends when a player has no more cards in his hand or the pack has been used up.

f. The winner is the player with the greatest number of matching cards on the board.

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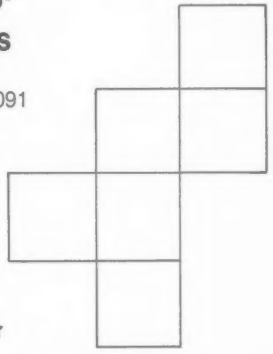

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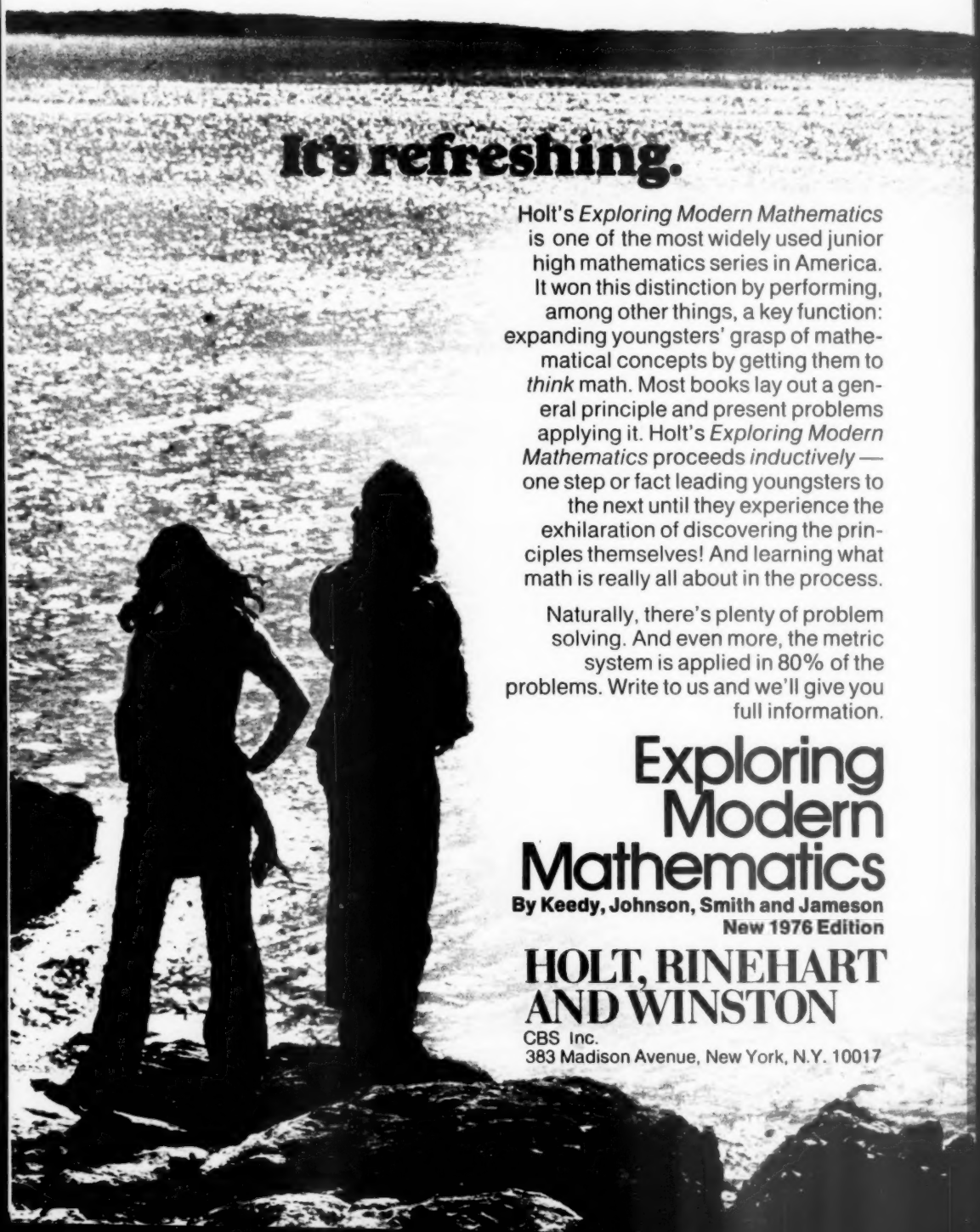

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Using flowcharts with first graders

SHERRY P. HUBBARD and
ROBERT B. ASHLOCK

At the time this article was submitted, Sherry Hubbard was working on the master of education degree at the University of Maryland. She is currently a first-grade teacher in Anne Arundel County, Maryland. Robert Ashlock is a professor of education at the University of Maryland, College Park, and director of the Arithmetic Center there.

As teachers work increasingly with individuals and with small groups, children need to be able to work independently. As a result, teachers need a method of communicating with independent workers, a method that does not require much of a teacher's attention. Primary teachers often find written communication difficult because the reading vocabulary of primary children is limited. Would it be useful to flowchart directions?

WHAT IS FLOWCHARTING?

A flowchart is a map of steps to follow in order to solve a problem. Computer programmers use the flowchart to map their program before preparing it on data cards for input into the computer. Symbols and shapes used in flowcharting must be defined just as symbols appearing on a map are defined. In the flowcharts presented in this article, shapes are defined as shown in figure 1.

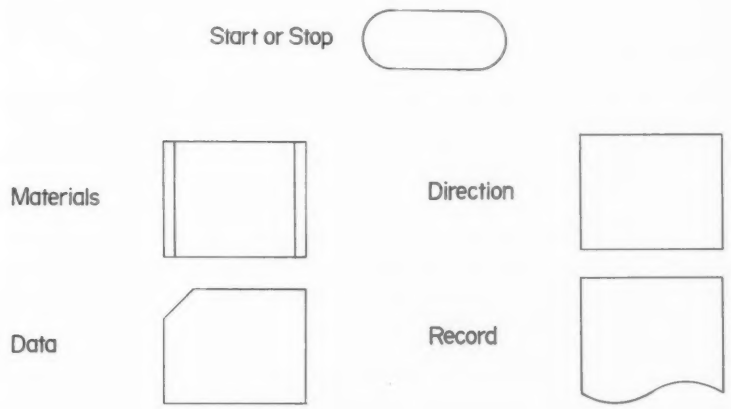


Fig. 1

The shape somewhat like an oval with the word *Start* indicates the beginning point. An arrow leads to the next step, another arrow leads to the next step, and so on. The material's shape tells what materials are needed to complete the flowchart. The data shape contains information a child is to consider while progressing through the flowchart. A direction that involves manipulation of objects or a thought process is indicated by the rectangle shape. The record shape is used when information is to be recorded on paper. An oval with the word *Stop* terminates the flowchart and indicates that the work is completed. Examples of flowcharts are shown in figures 2–9.

WHY USE A FLOWCHART?

Several reasons for incorporating flowcharts into instructional activities can be cited.

1. A teacher is forced to carefully evaluate the problem and determine each of the individual steps.
2. The flowchart defines each step clearly and isolates it from every other step.
3. The arrow indicates where to proceed next in easy-to-understand language.
4. The shape of the box communicates the nature of the process to be carried out and does so without written vocabulary.
5. Children like games and a flowchart resembles a gameboard.
6. Children are intrigued with computers and a flowchart is a tool for the computer.
7. Children like shapes and puzzles and a flowchart uses the ability to distinguish between shapes.
8. Children like to be independent and a flowchart can help them to be independent.
9. Children have free time and a flowchart can direct them to worthy uses of their free time.
10. Primary grade children have a limited reading vocabulary and a flowchart uses a minimum of words.

FLOWCHARTS IN THE FIRST GRADE

The flowcharts described in this article were used in a first-grade classroom. Each flowchart was printed on a manila folder and contained directions for one specific activity. The flowcharts were classified into eight categories: linear measurement, patterns, numerals, tens and ones, attributes, ordering, missing sums, and missing addends.

Several group lessons were conducted to familiarize the children with the format of flowcharts. Then for practice, the children worked in groups of four. Each group had a captain who was to answer questions and raise his hand for the teacher whenever additional help was needed. After the practice sessions children worked with the flowcharts individually.

A child selected a category, then picked out one flowchart. On completion of the flowchart he replaced it, then chose another flowchart from the same category or from another category. As a child worked with a flowchart, he recorded all work on a separate sheet of paper so the flowchart could be used again by another child.

A NUMERAL FLOWCHART

Consider the flowchart in figure 2. This flowchart focuses on the numeral and number word for a set of seven. The child would start; get a paper, pencil, and crayons; record his name on the paper; think about the data (7); record seven things on the paper; and stop. He would then put his paper in the place for finished work and return the flowchart to its proper place.

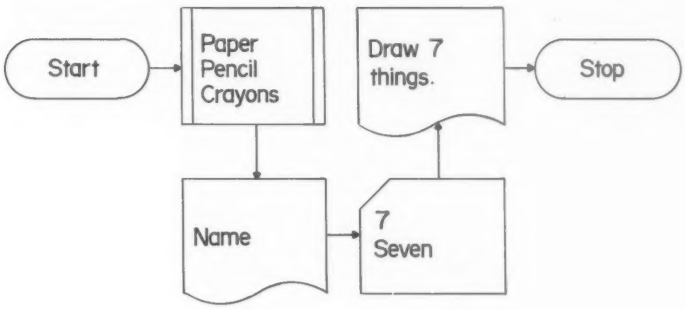


Fig. 2.

A TENS AND ONES FLOWCHART

An example of a tens and ones flowchart is found in figure 3. In this flowchart a child isolates groups of tens. A child would start; get paper, pencil, and crayons; record his name on the paper; think about the data (24); draw 24 things on his paper; circle two groups of ten; and stop.

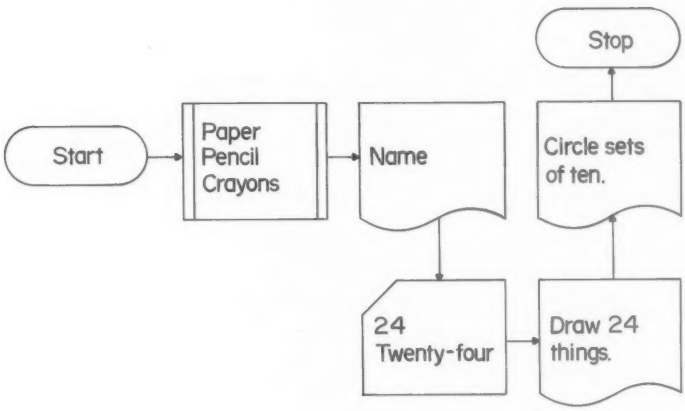


Fig. 3

A PATTERN FLOWCHART

Figure 4 is an example of a flowchart that helps children develop skill in recognizing patterns. The data for this flowchart was to be found in a box labeled Box 210. A child would take the box to his seat and make a pattern with the pieces he found in the box. If the box contained red and blue blocks, he might make a pattern of red-blue-red-blue-red-blue or red-red-red-blue-blue-blue, and so on. He would then draw the pattern he made on his sheet of paper.

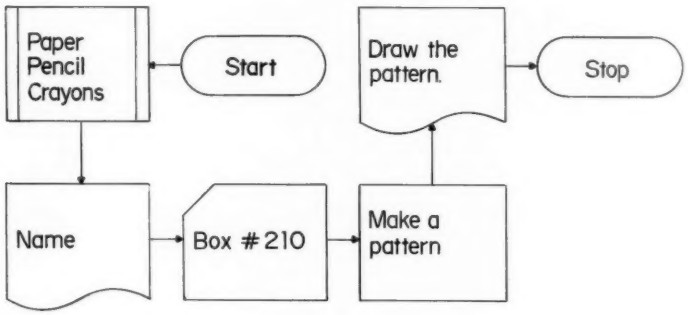


Fig. 4

A FLOWCHART FOR ORDERING

The flowcharts for ordering require a child to manipulate objects in a specified order. Figure 5 is an example of such a flowchart. A child would find the data in Box 312. He would order the pieces as directed, then record his order on his paper.

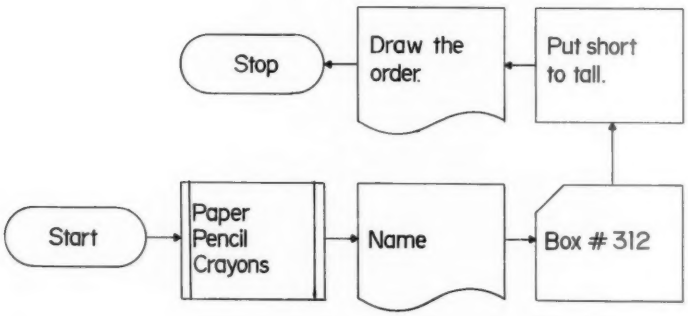


Fig. 5

AN ATTRIBUTE FLOWCHART

Consider the attribute flowchart in figure 6 in which a child's attention is drawn to the characteristics of an object. This flowchart merely requests a child to use the data in drawing some objects that have the mentioned attribute.

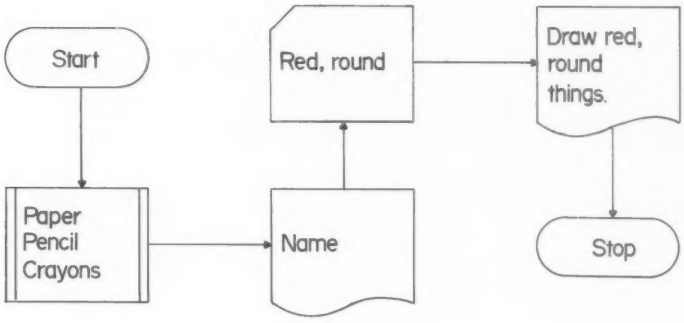


Fig. 6

A LINEAR MEASUREMENT FLOWCHART

The measurement flowcharts give a child the opportunity to investigate measurement with different tools. An example of a measurement flowchart appears in figure 7. A child finds the red stick and measures the mentioned object. Similar flowcharts direct a child to measure the same object using a longer or a shorter stick of a different color.

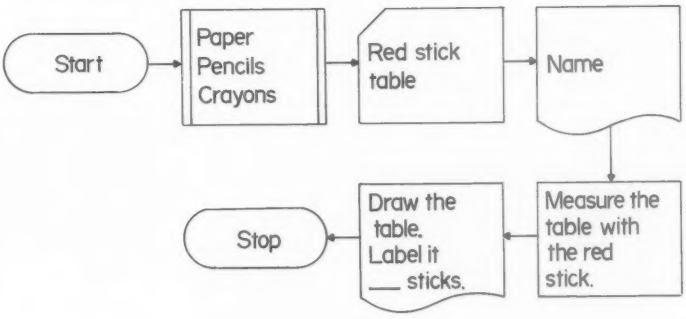


Fig. 7

A MISSING SUM FLOWCHART

Consider figure 8 for an example of a missing sum flowchart. A child is given two addends and asked to find the sum. This flowchart helps a child follow a specific sequence in solving an equation with the sum missing.

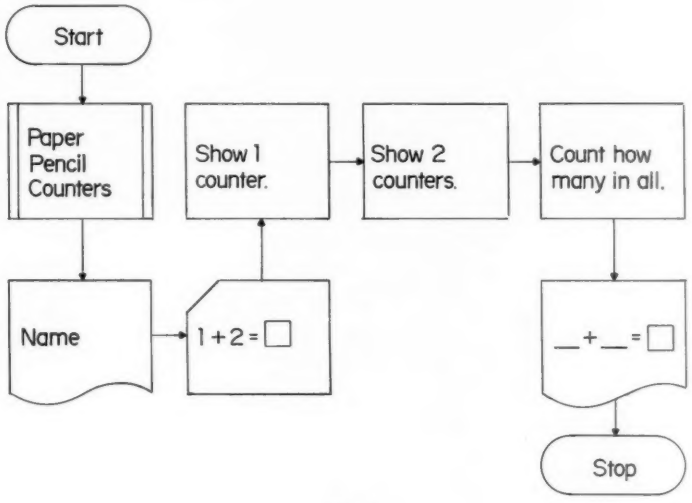


Fig. 8

A MISSING ADDEND FLOWCHART

In the missing addend flowcharts, the sum and one addend are given and the child is asked to discover the missing addend. One such flowchart is presented in figure 9.

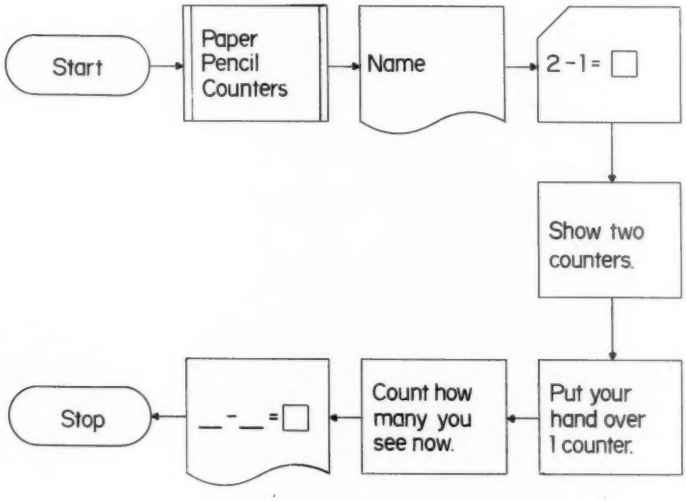
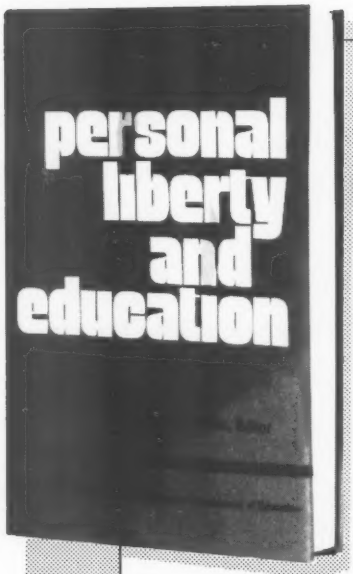


Fig. 9

CONCLUSIONS

The children seemed to enjoy using the flowcharts. In fact, many worked through one right after another. They were able to select and complete their work individually, though some help with reading vocabulary was required. Most children preferred working individually to working in groups of four.

The use of flowcharts by first graders is an idea that can be extended in many ways. Flowcharts can easily be adapted to other topics in mathematics and to other subject areas. Children sometimes want to make their own flowcharts or make class flowcharts.



personal liberty and education

Edited by Monroe D. Cohen
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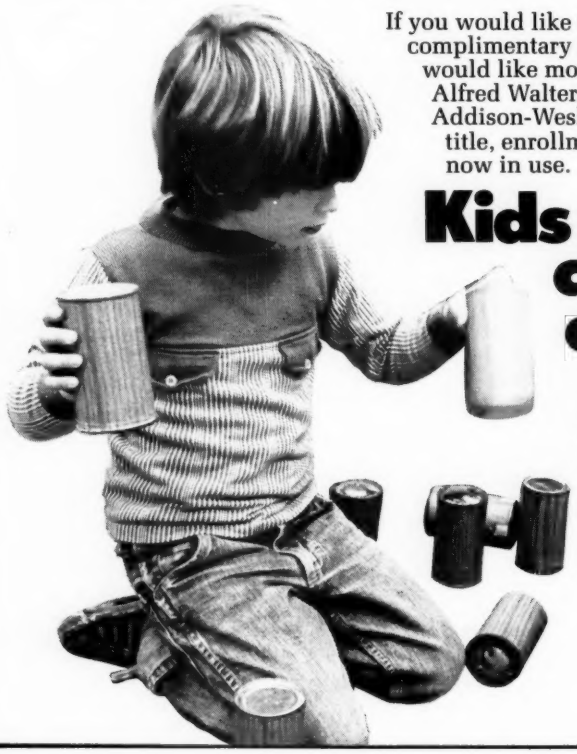
For understandable math concepts within the limits of their experience — **BASIC CONCEPTS OF MATHEMATICS FOR ELEMENTARY TEACHERS by Roy Dubisch.**

Professor Dubisch, the elder statesman of mathematics education, simplifies one of the most difficult aspects of teaching elementary mathematics — getting down to the student's experience level. Using actual material from elementary math books and articles which have appeared in *The Arithmetic Teacher,* Dubisch vividly illustrates the fundamentals prospective teachers will be dealing with. He urges them to remember, "a definition sums up an experience and should not precede it." Consequently, a child's comprehension of the real world must be considered when mathematics activities are presented. The role of discovery is emphasized by minimizing formal definitions and stressing learning activities. The author adapts devices like stick bundles, abaci, and geoboards for use with children. And for the prospective teacher — a thorough review of arithmetic skills including functions, binary operations, probability, statistics, and number theory.

An activities approach to teaching elementary mathematics is provided in the **MATHEMATICS-METHODS PROGRAM by John F. LeBlanc, Donald R. Kerr, Jr., and Maynard Thompson.** This is a system of instruction for prospective elementary school teachers which combines content and methods in a *learning* by *doing* atmosphere. Proven strategies for teaching each concept and an accurate perspective of today's elementary mathematics curriculum are given. The units are: Numeration, Addition and Subtraction, Multiplication and Division, Rational Numbers with Integers and Reals, Awareness Geometry, Transformational Geometry, Analysis of Shapes, Measurement, Graphs: The Picturing of Information, Number Theory, Probability and Statistics, and Experiences in Problem Solving.



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what's going on...

Edited by Jane Donnelly Gawronski, *Department of Education, San Diego County, San Diego, California*

HOME-SCHOOL MATH is the focus of a Title IV, Part C, grant in the St. Cloud school district. This project emphasizes materials for use by parents of children in grades K through 6 to help their children learn mathematics. Over 200 game-oriented activity kits for home or school use have been documented. The materials are available at cost. For additional information, contact Judith A. Maethner, Title IV Arithmetic Project, Seton Hall, 1204 Seventh Street South, St. Cloud, MN 56301.

* * * *

CLINICAL LEADERS IN MATHEMATICS EDUCATION (CLIME) is an innovative thrust in the use of supportive personnel for the improvement of mathematics performance in the elementary schools of Buffalo. It establishes a mathematics clinic within each of the city's six districts. CLIME personnel focus their support functions in two general areas of elementary mathematics, one on the diagnosis/remediation of children with learning difficulties and the other on the improvement of developmental instruction that may contribute to reducing the need for the former. For further information, contact Susanne M. Reeder, Supervisor of Mathematics, Buffalo Public Schools, Buffalo, NY 14202.

* * * *

MATH CIRCUIT RIDERS conduct classroom enrichment lessons and provide consultant help and information about mathematics materials to teachers in the small rural schools of eastern Oregon's Harney and Lake counties. The two contiguous counties stretch across nearly 20 000 square miles of sparsely populated high desert--an area roughly the size of Vermont and New Hampshire combined. In each county a mathematics circuit rider travels to the many widely separated one- and two-room schools serving some 3 000 school children scattered over the region. They conduct demonstration mathematics lessons at teachers' request, emphasizing laboratory and problem solving activities; consult with teachers about special learning problems of students; distribute laboratory and enrichment materials from central mathematics resource centers; and lead evening and Saturday inservice workshops. The project is supported by the two county intermediate education districts and by the National Science Foundation (NSF) through a grant from the Oregon Mathematics Education Council. More information is available from the Oregon Mathematics Education Council, 325 13th Street NE, Room 301, Salem, OR 93701.

SYSTEMATIC TEACHING AND MEASURING MATHEMATICS (STAMM) was designed to provide continuous progress in mathematics for the entire school experience of all students, grades K through 12. A framework of objectives and well defined processes of diagnosis, instruction, and assessment are provided. Careful monitoring of a student's progress, measurement of minimum competencies, and the promotion of alternative courses at the high school level are features. Teacher resource packets and guides are available. Objectives and criterion tests are furnished without cost to districts that adopt the program. A sampler kit of materials is available, including a slide-tape presentation. A commitment to teach and measure mathematics by a continuum of objectives is needed to take full advantage of the process and products offered. STAMM was validated for national dissemination by the joint panel of the U.S. Office of Education (USOE) and the National Institute of Education (NIE) in June 1976. For additional information, contact Cathy Peavler, Project Specialist, or Glyn H. Sharpe, Mathematics Coordinator, Jefferson County Schools, 1209 Quail Street, Lakewood, CO 80215.

* * * *

THE METROPOLITAN DETROIT METRIC COUNCIL (MDMC) is a volunteer organization of individuals representing the industrial, educational, publishing, retail, mass media, construction, and governmental sectors of the metropolitan Detroit area who wish to promote the use of the metric system and to provide coordination, guidance, and uniformity of metric information for the public in order to effect a smooth transition to the metric system. For additional information, contact Robert Williams, Lawrence Institute of Technology, Room 111, Administration Building, 21000 W. 10-Mile Road, Southfield, MI 48075.

* * * *

MATH-LAB CURRICULUM FOR JUNIOR HIGH, a Title III project, is now in its third year of field testing. Over 2 000 junior high students in nine Montana communities have used the project materials. A complete set of student performance objectives was developed and instructional packets were written to support these objectives. Each packet contains (1) activities for concept development, (2) problem solving materials, (3) alternatives to drill and practice, (4) content related games, and (5) calculator materials. Project evaluation during the initial two years indicates significant gains in student problem solving ability, computational scores that are generally higher than in previous years, and a positive student attitude toward the use of the laboratory approach. Additional information can be obtained through Daniel T. Dolan, Project Director, Columbus High School, Columbus, MT 59019.

Compound subtraction—an easier way

WILLIAM J. WENNER

A former elementary teacher and elementary mathematics chairman of the Wellsboro School District, William Wenner is currently principal of the Don Gill Elementary School in Wellsboro, Pennsylvania.

In order to perform compound subtraction, pupils are generally taught to use one of three methods, the decomposition, the equal-addition, or the additive. (For a concise explanation of these techniques see W. G. Quast's article "Method or Justification?" in the *Arithmetic Teacher,* December 1972, pp. 617–22.)

These three methods assume that pupils already have a command of the 100 basic addition and subtraction facts, a supposition which, with regard to young children, is not necessarily borne out by teaching experience. Watching a child in the act of performing compound subtraction will often reveal the cause of difficulty. It is not always due to the lack of understanding the algorithm. More often, it is due to the inability of the pupil to handle with assurance the thirty-six number pairs having minuends greater than ten.

An oversimplification of the problem might suggest that what is needed, in such a case, is for the teacher to provide a variety of activities that will promote habituation of the unlearned facts. Presto! Johnnie's ready for compound subtraction. But, if Johnnie hasn't learned those facts through the variety of teachers and activities he's had so far, then maybe he'll still have difficulty learning them. If he is to do compound subtraction soon, Johnnie had better be taught another way that avoids for now his big difficulty. What Johnnie needs is an easier way. And for the slow memorizers, that's the "base subtraction" method.

This method depends on the memorization of only those number pairs having sums or minuends of the base or less, and on the understanding of place value and grouping in sets of the base. Thus, in base ten, it avoids for subtraction purposes the need to memorize the more difficult set of facts that Grossnickle, Brueckner, and Reckzeh (1968) classify as "Set III" of the basic addition and subtraction facts.

Using this method, a pupil faced with the example

$$\begin{array}{r} 527 \\ -189 \\ \hline \end{array}$$

would be taught to use the following thought pattern:

> I can't take 9 ones from 7 ones, at least in the set of whole numbers, so I'll rename the 2 tens of the minuend as 1 ten plus 10 ones and then subtract the 9 ones of the subtrahend from the 10 ones, add the remainder 1 to my 7 ones of the minuend, and write the sum 8 as a remainder.
>
> I can't take 8 tens from 1 ten, so I'll rename the 5 hundreds of the minuend as 4 hundreds plus 10 tens, subtract the 8 tens of the subtrahend from the 10 tens, add the remainder 2 tens to the 1 ten of the minuend, and write the sum 3 as my remainder.
>
> I subtract the 1 hundred from the 4 hundred and write the remainder 3 in the hundred column.
>
> The answer is 338.

Note how this has circumvented the need to subtract 9 from 17 and 8 from 11.

To someone already familiar with another technique, the thought process change involved in the "base subtraction" method may not appear at first sufficiently significant to warrant a change in teaching methods. But to fully appreciate the problem faced by slow memorizers, try to do a few subtraction examples in an unfamiliar base yourself. (You need not have Johnnie subtract in any base but ten.) This experience will somewhat parallel that of the child when he attempts to do compound subtraction without a ready command of the more difficult number facts. First, try these examples using your usual method, and then try the base subtraction method.

732_{eight}	321_{five}	512_{six}
-564_{eight}	-143_{five}	-321_{six}

Which did you find easier? And so will your slow memorizers, who will be able to do compound subtraction with such speed and accuracy using the base subtraction method, that you won't believe it's the same Johnnie doing the work.

References

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Smith, Robert F. "Diagnosis of Pupil Performance on Place Value Tasks." *Arithmetic Teacher* 20 (May 1973):403–8.

M & M's candy: a statistical approach

Surely you have wondered which colors are most predominant in a package of M & M's candy. Well, why not use a statistical approach in your classroom to research this topic.

I tell my class that each student will have to analyze an individual "data package" by color. Students work in teams (row, group, table), but every student must complete a frequency-distribution chart, as shown in table 1, and draw histograms for the results in columns *a*, *c*, and *e*. I enjoy the change of expression when the students are finally given the "data package" and told, "When through with your records, you may dispose of your data in some appropriate manner."

After the data have been collected, there are many applications for it—determining the most probable proportions of each color, discussing deviations with respect to sample size, normal distribution, and other concepts. We also have taken a poll on the most favorite and least favorite "flavors," with the intent of establishing ideal proportions consistent with our results.

If you are curious, our research indicates that dark brown is the most prominent color, but I suppose you sensed that anyway.

Color of item	*a* Frequency in your own package	*b* Frequency totals for your group	*c* Average frequency for your group	*d* Frequency totals for the class	*e* Average frequency for the class
Red					
Orange					
Yellow					
Green					
Tan					
Brown					

DONALD W. HYATT
Stephen Foster Intermediate School
Fairfax County, Virginia

Know why I'm so good at math? I've had a lot of practice.

Oh, not just playing games. I mean in school math lessons. Even before our teacher teaches, we do "Getting Ready" pages — they're kind of like warming up before a game. We *practice* the skills we'll need for the new lesson. Now comes the teaching. Then we *practice* the new skill using the exercises on the lesson page. Then we *practice* some more in the workbooks and on the duplicated sheets. And running through our book are even more *practice* opportunities: "Maintaining Skills," "Games," "Just for Fun," "Try This," "Sharpshooting" (without pencil and paper!), "Extra Practice," and "Extra for Experts." Of course, not everyone does all these exercises. We each do as many as we need to, to be really good. Sometimes I do them all because it's fun.

Houghton Mifflin might not have a corner on practice, but they've sure provided plenty of it.

SCHOOL MATHEMATICS
CONCEPTS AND SKILLS
K-8

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Property games

JOHN C. PETERSON and
GENE DOLSON

For seven years John C. Peterson taught content and methods courses for prospective elementary and secondary school teachers at Eastern Illinois University. He is currently working on the Metric Education Project at the Center for Vocational Education, The Ohio State University, Columbus. Gene Dolson is associate professor in mathematics and education at Eastern Illinois University, Charleston. He teaches content and methods courses for elementary teachers and supervises secondary student teachers in mathematics.

Property blocks are often used in kindergarten and first grade to help teach children to distinguish shape, color, size, and thickness. Various forms of bingo are also used by teachers in other grades to provide interesting ways for students to practice various mathematical concepts. Shape-o and Count Shape-o are *kindergarten level* games that combine aspects of Property Blocks, numeration, and bingo. They can be played by children in the kindergarten.

SHAPE-O

The properties used in Shape-o are color and shape. The use of five colors (black, blue, green, red, and yellow) and four shapes (circle, rectangle, square, and triangle) provide twenty different color-shape combinations. Each of these twenty combinations is drawn on a square of paper or on one side of a block. These paper squares or blocks will be referred to as the "call cards."

Game cards are made by duplicating on 5-by-8-inch file cards the "playing field" shown in figure 1. Each game card consists of a 4-by-4 array of squares. On each game card the teacher draws, with permanent ink pens of various colors, sixteen of the twenty color-shape combinations, as shown in figure 2. None of the color-shape combinations should appear more than once on each card, and the sets of combinations should be randomly made.

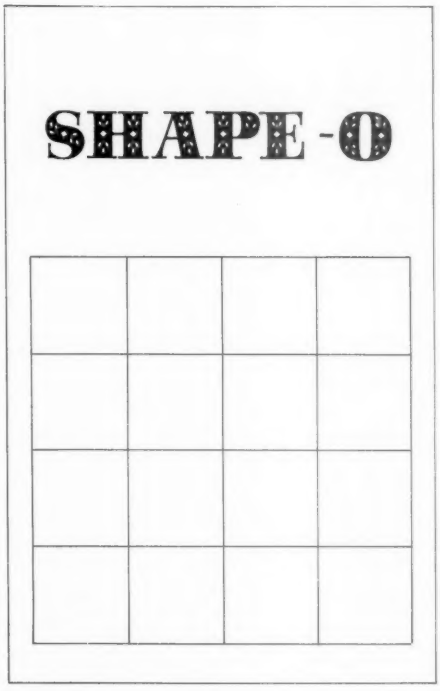

Fig. 1

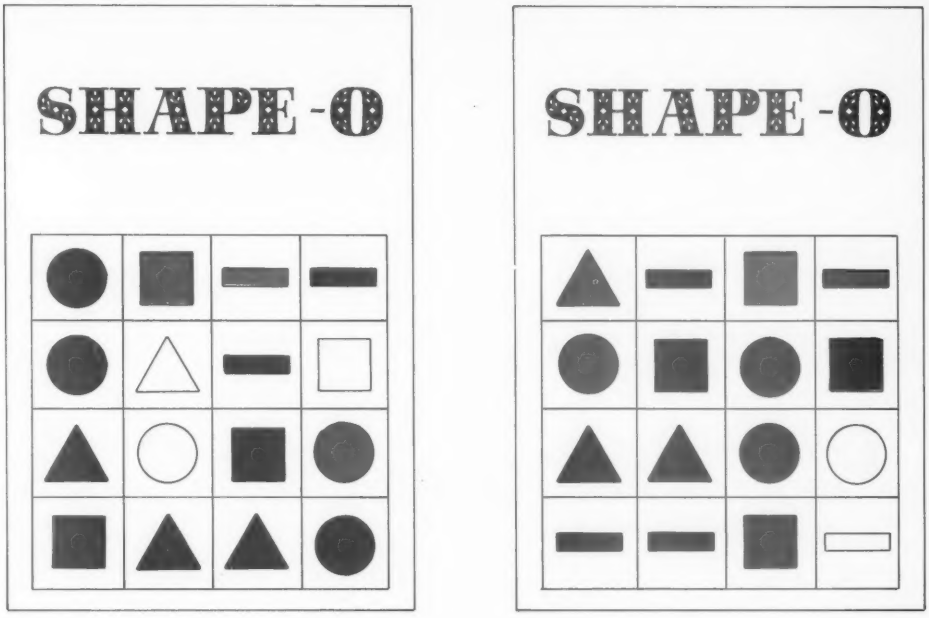

Fig. 2

In order to play the game each student will need a game card and a supply of markers to cover items that are called. The call cards should be placed in an open container. The teacher holds the container of call cards while a student draws a call card and then describes the object to the class; for example "green circle." Students who have a green circle on their game card place a marker on the green circle. A second student draws a call card and describes it to the class; for example "red square." Students with a red square on their game card place a marker over the red square. The game continues until a student gets four markers in the same row, same column, or one of the major diagonals.

Since there are twenty color-shape combinations and only sixteen playable spots on each game card, not every student will be able to place a marker on his card after each call card is drawn. Students will be able to place a marker on their card about 80 percent of the time. Furthermore, the columns are not labeled as in regular bingo. Thus, when "green circle" is called, it can appear any place on the game card.

COUNT SHAPE-O

The properties used in Count Shape-o are color, shape, and quantity. The use of two colors (red and green), three shapes (circle, square, and triangle), and five quantities (one through five) provide thirty different color-shape-quantity combinations. As in Shape-o, each of these thirty combinations is drawn on a square of paper or on one side of a block. These paper squares or blocks will again be referred to as the "call cards."

Count Shape-o cards are made the same way as Shape-o cards. File cards of different colors may be used to readily distinguish the two sets. Since there are thirty color-shape-quantity combinations and only sixteen playable spots on each game card not every student will be able to place a marker on the game card after each call card is drawn. Students in Count Shape-o will be able to place a marker on their card

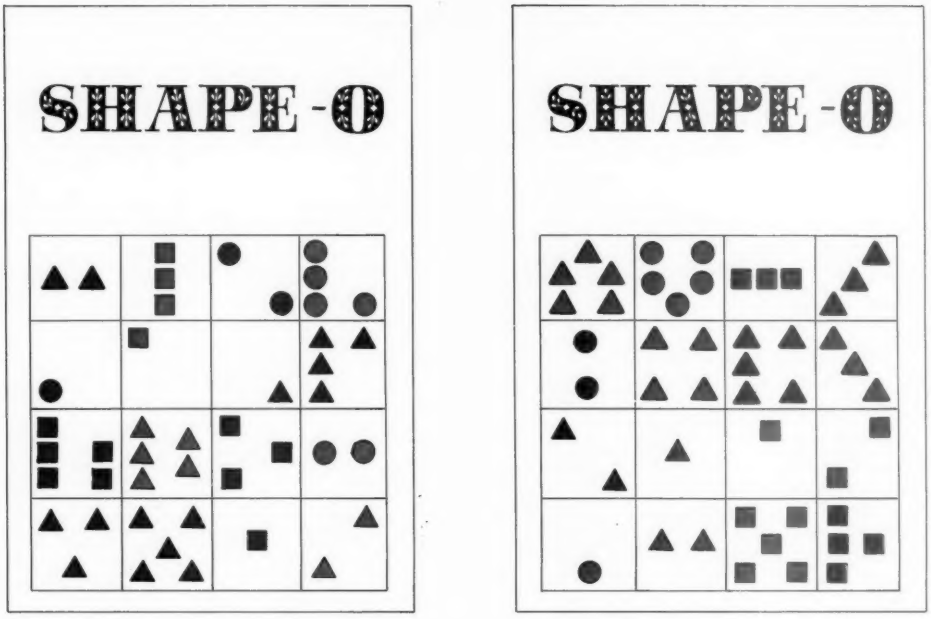

Fig. 3

about 60 percent of the time, as compared to the 80 percent for Shape-o. Sample Count Shape-o cards are shown in figure 3.

In order to play the game each student will need a game card and a supply of markers to cover items that are called. Call cards are placed in an open container and students take turns selecting call cards and describing the selected card to the class; for example, "two red circles" or "five green squares." Notice that a color-shape-quantity combination does not always appear in the same manner on the playing cards. Thus, three red circles may be arranged vertically on one playing card, horizontally on a second, and triangularly on a third. Students will not be locked into any one particular arrangement for a given quantity.

By using different colors, different shapes, different quantities, or by labeling the columns, different versions of Shape-o and Count Shape-o can be enjoyed by young children.

TEAMWORK SUPPORTS THE PROFESSION

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A fun way to introduce probability

RICK BILLSTEIN

An assistant professor in the department of mathematics at the University of Montana in Missoula, Rick Billstein is involved in elementary school teacher training at the university. This article is a result of some work in probability, which the university requires of all prospective elementary school teachers.

The typical introduction to probability usually involves tossing a single coin. The students and teacher will soon agree that this is a trivial activity. The possible outcomes of the experiment are a head, H, or a tail, T. The outcome set could simply be listed as {H, T}. If the coin were tossed a large number of times, the proportion of heads appearing should approach 1/2. For this reason we assign to each outcome the probability 1/2. We also say that the two outcomes are *equally likely* and the coin is *fair*. Figure 1 illustrates another way of describing one toss of a fair coin. This description will hereafter be referred to as a *tree diagram*. Each branch corresponds to an element in the outcome set with its appropriate probability labeled on the branch. Also notice that the sum of the probabilities of all the outcomes in the set is equal to 1.

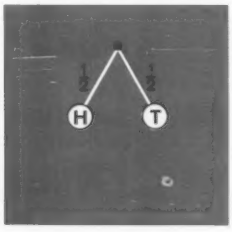

Fig. 1

Suppose a coin is tossed twice. The possible outcomes of the experiment are a head followed by a head, HH; a tail followed by a tail, TT; a head followed by a tail, HT; or a tail followed by a head, TH. Most students will have very little trouble listing these possible outcomes in a set, {HH, HT, TH, TT}. After much experimenting and discussion, the students will notice that the probability 1/4 can be assigned to each of these outcomes. Figure 2 shows a tree diagram of the experiment.

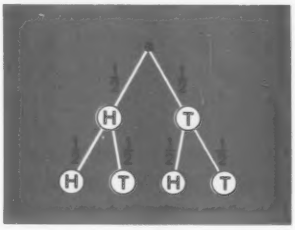

Fig. 2

Notice that the probability assignment to each element in the outcome set is simply the product of all probabilities along the branches of the path. For example the probability of HH is $1/2 \times 1/2$, or 1/4. The author will not attempt to motivate this idea any further but simply state that the probability of any path is equal to the product of all probabilities along the branches of the path.

With this background the students are ready for a new coin game. Let the students choose one of the four equally likely outcomes listed from tossing the coin twice. The teacher then chooses a different outcome. The coin is flipped until either the

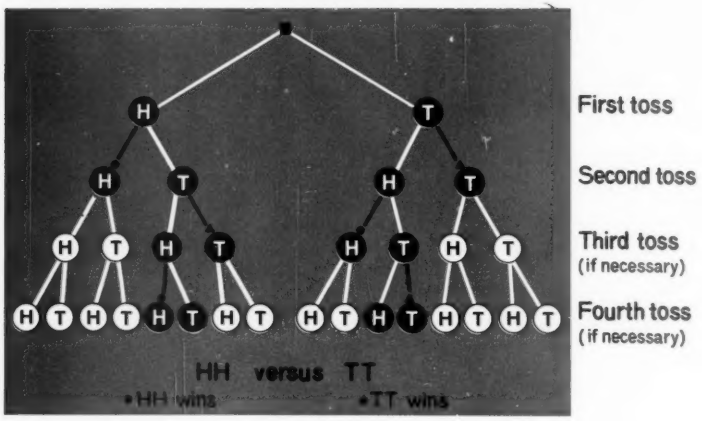

Fig. 3

students' choice or the teacher's choice appears. Suppose, for example, that the students choose TT and the teacher chooses HT. If the first two flips show TH, no one has won and the game continues. If after six flips, the string THHH*HT* is obtained, the teacher is declared the winner since the sequence HT finally appeared. Suppose the students choose HT and the teacher chooses TH. If the first two tosses show HH, no one has won and the game continues. If after three flips the sequence H*HT* appears, the students win because their choice, HT, has appeared in the sequence.

To develop a strategy for playing this game we might first examine the choices HH versus TT. If we refer to the tree diagram (fig. 3) and use a simple argument involving symmetry, it is easy to convince students that this is a fair game. A win is possible on every toss for both players.

Now consider HH versus TH. This at first glance also appears to be a fair game. However, observe the tree diagram (fig. 4) for this case. On the second toss, notice that the teacher's, TH, and the students', HH, outcomes each appear exactly once. At this point students may still feel that this is a fair game. Careful analysis will now show that if three or more tosses are necessary, the students' choice, HH, will never appear. Since the students' probability of winning

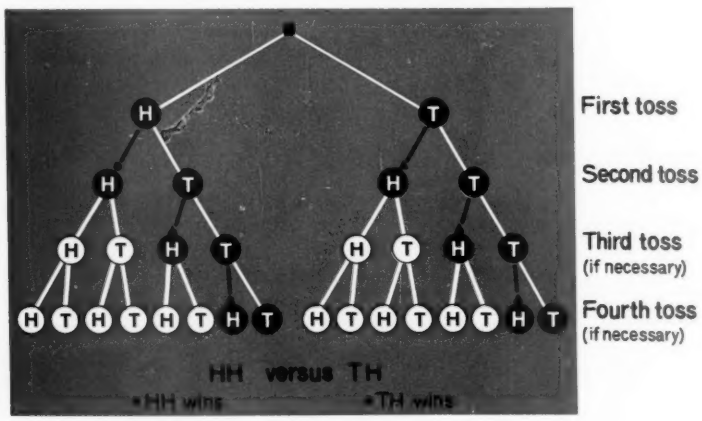

Fig. 4

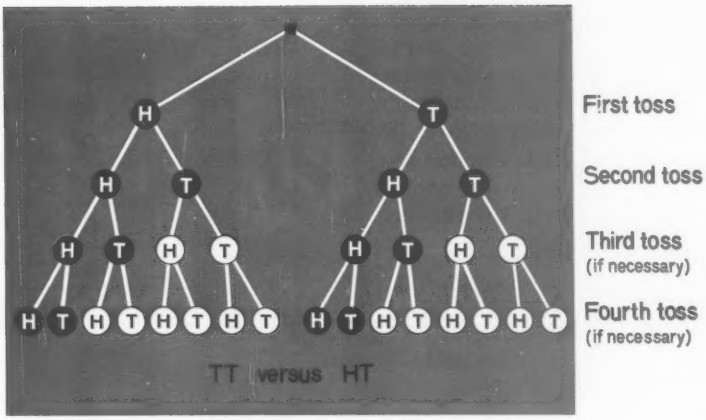

Fig. 5

has now been shown to be 1/4, and since the sum of the probabilities must be one, the teacher's probability of winning must be 3/4. Hence the game is no longer considered fair. A similar argument holds if the students' first choice is TT and the teacher's choice is HT. (Fig. 5)

The probabilities of the teacher's winning the game are summarized in the table in figure 6. Of the twelve possible games, only eight are fair in the sense that both choices are equally likely.

Teacher's choice (second)

Students' choice (first)	HHT	THT	TTH	TT
HHT	–	.50	.75	.50
THT	.50	–	.50	.25
TTH	.25	.50	–	.50
TT	.50	.75	.50	–

Fig. 6

After the details of the game involving two coins are understood by all the students, the better students might be encouraged to investigate the corresponding game involving three flips of a coin. If a coin is flipped three times there are eight equally likely outcomes—HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT. If, for example, the first choice is HHT, is there a second choice that has probability greater than 1/2 of winning? Is there a second choice that will make this a fair game? Is there a second choice that has the best chance of winning? The interested reader may find this game discussed in the Winter 1973 issue of *The Two-Year College Mathematics Journal* (Craswell).

A second game that may be used in the same way is a game called "Win" (Smith 1973). This game is played with the nonstandard dice as shown in figure 7. The students again will be given first choice of any one of the four dice. The teacher then chooses one of the remaining dice. The two selected dice are then rolled at the same time. The player with the higher number is the winner.

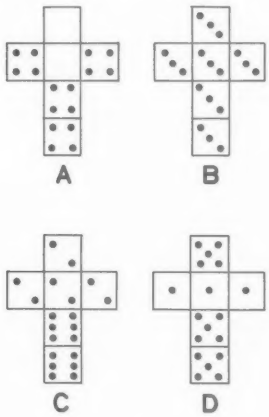

Fig. 7

This again is a surprising game. Suppose the students choose A and the teacher chooses D. Figure 8 shows that the teacher would win 24 out of 36 possible rolls for the two dice. As you will soon discover, no matter what choice the student makes first, the teacher may always make another choice that has probability 2/3 of winning:

Students' choice (first)	Teacher's choice (second)
A	D
B	A
C	B
D	C

These results can be substantiated by examining the outcome sets for each case.

Probability can be a fascinating study for elementary students. It can be a subject of interest to the better students as well as to the slower students because the problems can be identified with games, models, or real-life situations. There are many problems where intuition can be stressed and there are also many cases such as those given in this article where intuition fails and a careful analysis is required. In the words

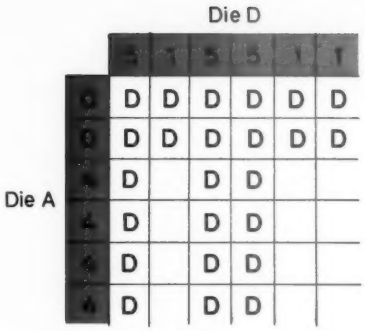

Fig. 8

of Freudenthal (1970), "probability applies in every day situations, in games, in data processing, in insurance, in economics, in natural sciences. There is no part of mathematics that is as universally applied, except, of course, elementary arithmentic."

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Craswell, Keith J. "An Interesting Penny Game." *The Two-Year College Mathematics Journal* 4(1973):18–25.

Freudenthal, Hans. *The Aims of Teaching Probability*. Stockholm: Almquist and Wiksell Vorlog AB, 1970.

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Polyhedron Models For the Classroom,
by Magnus J. Wenninger
Revised edition of popular NCTM book reprinted six times in eight years.

The second edition features detachable center-fold sheets of **full-scale drawings** from which templates for polyhedron models may be easily and accurately made. Fascinating enrichment activity for students of geometry. Combines mathematics and artistic creativity; results in beautiful ornaments or mobiles for the classroom. 80 pp., **$1.60*, 1975.**

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Activities that contribute to the student's personal understanding of key concepts in mathematics

IDEAS

Prepared by **George W. Bright,** *Northern Illinois University, DeKalb, Illinois*

Each IDEAS presents activities that are appropriate for use with students at various levels in the elementary school. The activity sheets are so arranged that they can be easily removed and reproduced for classroom use. Permission to reproduce these worksheets for classroom use is not necessary.

IDEAS for January features activities with fractions. The primary-level activities provide opportunities to develop the concept of a fraction and the intermediate-level activities emphasize computation with fractions.

Again, "thank yous" go to Millie Lewis, Betty Pullin, and Candy Slitor in the DeKalb County (Georgia) Schools and to all their students who helped field test these activities.

For Teachers

Objective: To reinforce the concept of one-half of an object

Levels: K, 1

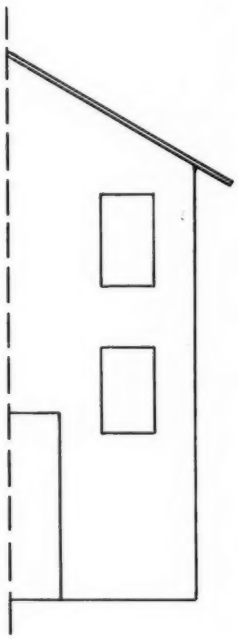
Directions for teachers:

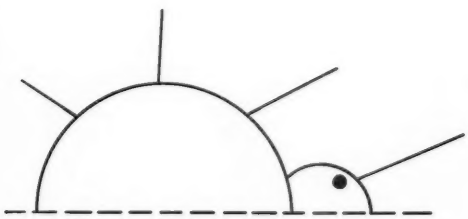
1. Give each student a copy of the worksheet.
2. Read the directions to the students.
3. When they have finished you might want to let them color the pictures.
4. Ask them to identify the bottom right-hand picture (for example, doughnut, bagel, tire, and so on).

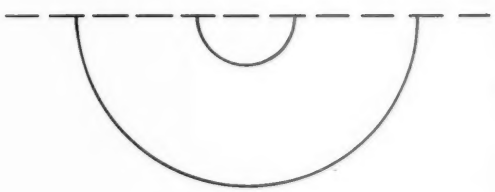
IDEAS

Name _____

Half of each object is pictured.
Draw the missing half.







From The Arithmetic Teacher

IDEAS For Teachers

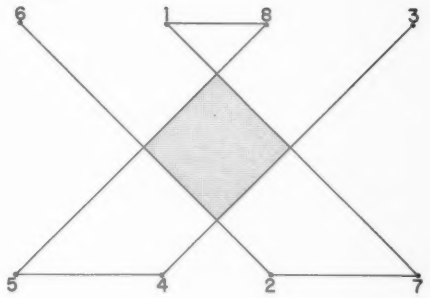
Objectives: To reinforce the concept of one-half of a set of objects

Levels: 1, 2, 3

Directions for teachers:

1. Give each student a copy of the worksheet.
2. Read the directions aloud.
3. Some students may want to draw a ring around each half of each set.

Answer:



Readers' Dialogue

Dear Editor:

In the February 1976 issue of *Arithmetic Teacher*, Myron Tepper presented a number pattern for the expansion of numbers ending in 9, for example,

$$19 = (1 + 9) + (1 \times 9),$$
$$29 = (2 + 9) + (2 \times 9),$$
$$\vdots$$

or in general

$$10n + 9 = (n + 9) + (n \times 9).$$

Mr. Tepper found similar patterns for numbers ending in 8 and 7, which he summarized with the identities

$$10n + 8 = (n + 8) + (n \times 8) + n, \text{ and}$$
$$10n + 7 = (n + 7) + (n \times 7) + 2n.$$

These identities can be extended indefinitely, for example,

$$10n + 6 = (n + 6) + (n \times 6) + 3n$$
$$10n + 5 = (n + 5) + (n \times 5) + 4n$$
$$10n + 4 = (n + 4) + (n \times 4) + 5n$$
$$\vdots$$

The coefficient of the last term in the expansion is the complement in 9 of the last digit of the given number. The expansion uses only the coefficient and the digits of the given number.

Any number can be expanded in this form, for example,

$$137 = (13 + 7) + (13 \times 7) + 2n$$
$$276 = (27 + 6) + (27 \times 6) + 3n$$
$$888 = (88 + 8) + (88 \times 8) + n$$
$$543 = (54 + 3) + (54 \times 3) + 6n$$
$$\vdots$$

Victor Mastrovincenzo
Bronx, New York

Name _____

Separate each group of objects into halves.
Count the number in one half and write your answer in the blank.
Connect the answers in order.
Color the square.

a. ○ ○ ○ / ○ ○ ○ _____

b. X X X X / X X X X _____

c. ∩ ∩ / ∩ ∩ ∩ ∩ / ∩ ∩ ∩ ∩ / + + + + / + + + + / + + + + _____

d. _____

e. ☆ ☆ _____

f. △△△△△△△△△△ / △△△△ _____

g. □ □ □ □ _____

h. * * * * * * * * * _____

6 1 8 3

5 4 2 7

IDEAS For Teachers

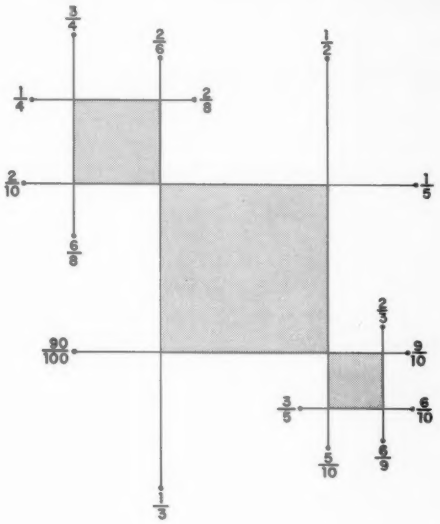
Objective: To practice identifying equivalent fractions

Levels: 3, 4, 5

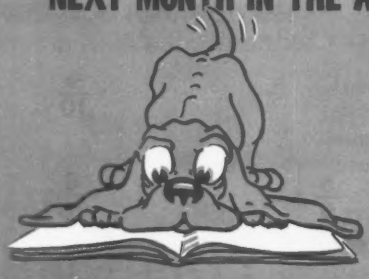
Directions for teachers:

1. Give each student a copy of the worksheet.
2. Read the directions with the students. The students will draw more accurate figures if they use a ruler or straightedge. The squares will be recognizable, however, if a straightedge is not used.
3. You may want to give an example of equivalent fractions before the students begin.

Answer:



NEXT MONTH IN THE ARITHMETIC TEACHER:



Look for—

- ways of using playing cards for drill, practice, and even concept development in arithmetic;
- ideas for using graphing as a means to predicting outcomes; and
- a descriptive report of arithmetic education in primary schools in Tokyo,

as well as the familiar sections.

Name _____

Use a ruler to draw a line segment between each pair of equivalent fractions.

Color the three squares.

$\frac{3}{4}$ $\frac{2}{6}$ $\frac{1}{2}$

$\frac{1}{4}$ $\frac{2}{8}$

$\frac{2}{10}$ $\frac{1}{5}$

$\frac{6}{8}$

$\frac{2}{3}$

$\frac{90}{100}$ $\frac{9}{10}$

$\frac{3}{5}$ $\frac{6}{10}$

$\frac{5}{10}$ $\frac{6}{9}$

$\frac{1}{3}$

From The Arithmetic Teacher

IDEAS For Teachers

Objective: To practice adding fractions whose sum is 1

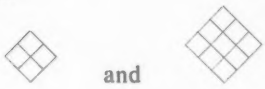
Levels: 5, 6, 7

Directions for teachers:

1. Give each student a copy of the worksheet.
2. Let the students read the directions. Emphasize that the students are looking for pairs of fractions whose sum is 1; for example, $\frac{3}{8} + \frac{5}{8}$ or $\frac{2}{3} + \frac{1}{3}$. The use of straightedges will make the drawings more accurate, but you do not need to insist on their use.
3. Three fractions, $\frac{2}{8}$, $\frac{2}{4}$, and $\frac{2}{10}$, are not in lowest terms. If students have trouble matching these fractions with

   $$\frac{3}{4}, \frac{1}{2}, \text{and } \frac{4}{5},$$

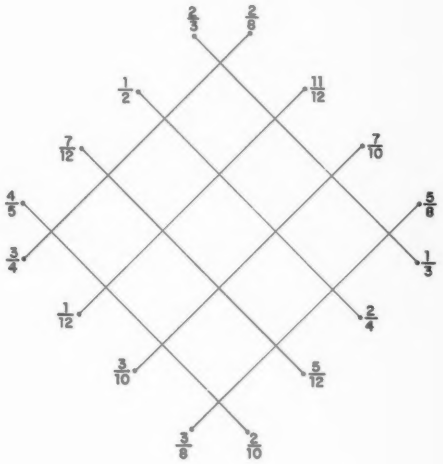
   respectively, suggest to the students that they rewrite all fractions in lowest terms before trying to match them.
4. After the students have drawn all the line segments, encourage them to identify not only the simplest squares; for example, ◇ ; but also more complex ones.



and

The total number of squares is 14.

Answer:



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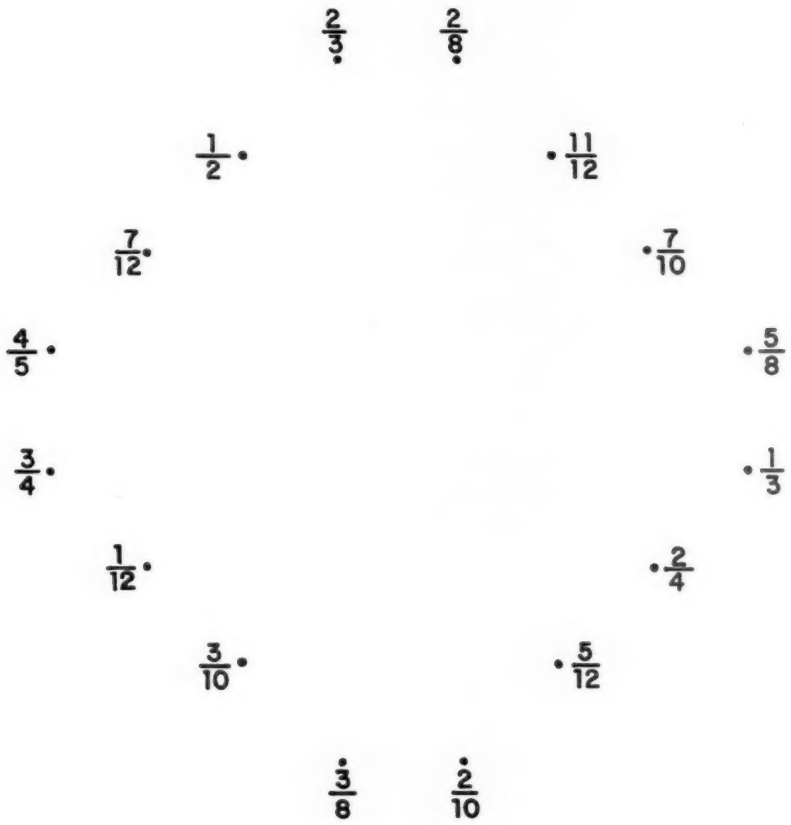
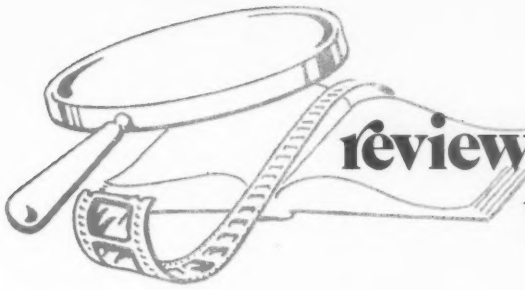
Name_____

Use a ruler to draw a line segment between each pair of fractions whose sum is 1.

Count all the squares



From The Arithmetic Teacher



reviewing and viewing

New books for pupils

Edited by **Edward C. Rathmell,** *University of Northern Iowa, Cedar Falls, Iowa*

"The Basic Skills in Math Series." A series of four books: *Beginning Addition & Subtraction: A*; *Beginning Addition & Subtraction: B*; *Beginning Multiplication & Division*; *Beginning Fractions.* Clara E. Clark and Gretchen B. Lytle. 1975, 128 pp. ea., $2.95 ea., Teacher's Manual, $3.25 ea. Laurel Publishing Company, 71 South Turnpike, Wallingford, Conn. 06492.

These books are designed to develop basic concepts and skills in the elementary mathematics curriculum. The materials have been organized so children can thoroughly learn one topic before beginning another. The use of inexpensive concrete models is also carefully built into the sequences to facilitate learning. The sequences progress through developmental levels, from concrete to representational to abstract.

The Teacher's Manual includes prerequisites for each topic; a description of materials that are needed; procedures, including suggestions for concrete experiences to precede the representational work; checkpoints for the individual learners; and practical mathematics activities. There are also spirit masters, checklists for evaluation and diagnosis, and supplementary activities and kits for available concrete materials.

It appears to the reviewer that these materials will prepare students for meaningful experiences with the computational algorithms; however, the materials themselves do not emphasize the algorithms. The emphasis is on conceptual development and building meaning for mathematical symbolism. The books are appropriately titled. They provide a meaningful beginning for each topic. Together with a few more experiences with the computational algorithms, these materials should enable children to master the basic concepts and skills.

This program could be very beneficial for students in a remedial situation. There is an abundance of activities that can develop meaning for the initial concepts and skills related to addition, subtraction, multiplication, division, and fractions. It could also be used to introduce topics in the regular mathematics program—addition and subtraction in the primary grades, multiplication, division, and fractions in the middle grades. However, it is not a complete program as it now stands. Also, the applications that include measurements make use of customary, not metric, units.—E. R.

50 Spirit Masters on Decimals and Percents. Linda Dritsas. 1976, $14.00. J. Weston Walch, Publisher, Portland, Me. 04104.

This set of masters is designed to provide a variety of paper-and-pencil practice activities for decimal and percent topics. Different types of puzzles are included for interest and sets of word problems are provided for relevance. The puzzles contain a good selection of coded messages, shaded messages, magic squares, magic stars, and so on, in an attempt to lessen the drudgery of practice. The word problems offer practical applications of decimals and percent, though most measurements are given in the customary system.

The 50 masters give broad coverage—from reading and writing decimals through applications of percent such as buying a home—and seem most appropriate for junior high and certain senior high school classes. Teachers will probably want to select those activities that are suitable for their particular students. Care should be exercised by the teacher to be sure that the learners understand that the term *decimal* refers to *decimal point* in some activities and to *decimal number* in others.

When used as a supplement to good instruction, these masters offer a variety of interesting practice activities for decimal and percent topics.—*Glenn Nelson, University of Northern Iowa, Cedar Falls.*

Basic Math Puzzles: 50 Spirit Masters. Robert S. Graflund. 1976, $14.00. J. Weston Walch, Publisher, Portland, Me. 04104.

These masters are designed to be used as a supplement in basic or refresher mathematics courses at the junior and senior high school levels. They attempt to provide motivation for learners with previously limited success in mathematics, short attention spans, and reading difficulties by the use of cartoons, coded messages, shaded messages, magic squares, crossnumber puzzles, and by keeping written instructions brief. Six of the masters are tests. There is a pretest, a posttest, and eight shorter tests over more specific topics, such as addition-subtraction of fractions.

The masters cover a number of different topics, from practice on basic operations with whole numbers, fractions and decimals, through percent, graphing, area and volume, exponents, the Pythagorean theorem, protractor use, and set operations.

It should be noted that no metric measures are used in this set, only the customary units. Also, some care is needed in reading mixed numbers. Since they are all printed horizontally, like 3 1/4, they could be misinterpreted as improper fractions, and vice versa, by less discriminating readers. A personal preference is for the less confusing form of $3\frac{1}{4}$.

These masters do not instruct, but they can provide interesting practice on skills. There is wide variety in both topics covered and in the types of puzzles used. They could be useful to the teacher looking for different practice activities for these topics.—*Glenn Nelson, University of Northern Iowa, Cedar Falls.*

Silver Burdett Essentials of Mathematics. Harry S. Couzins and Alice A. Huffman. 1976, 28T + 360 pp., $6.36; Teachers' edition, $8.85. Silver Burdett Company, 250 James Street, Morristown, N.J. 07960.

This is a remedial textbook for students with deficiencies in computational skills. It is appropriate for junior high and some senior high students. It is designed to meet the needs of slow readers.

The book contains a well-chosen variety of games, activities, and applications that are appropriate to the computational objectives. The teacher's edition provides objectives, answers, and suggestions for the lessons.

The topics include addition, subtraction, multiplication, and division of whole numbers, fractions, and decimal fractions. It also includes ratio and percent and a chapter on geometry.

This book should help motivate and provide the experience with basic skills that students with computational deficiencies need.—*Joe Hohlfeld, Iowa Area Education Agency VII.*

New books for teachers

Edited by **Edith Robinson**, *Asheville, North Carolina*

For the content course

Modern Elementary Mathematics. Malcolm Graham. 1975, 449 pp., $10.95. Harcourt Brace Jovanovich, Inc., 757 Third Avenue, New York, N.Y. 10017.

A well-written, formal treatment of the foundations of elementary school arithmetic. The final three chapters are devoted to geometry (two chapters, 52 pages) and to probability (24 pages). Logic and reasoning appear in an appendix.

Both content and treatment will be familiar to experienced instructors of content courses. The exposition is clear and unambiguous, the parts fit into a consistent whole, and the exercises are routine but thorough. In particular, the treatment of sets has been excised, with special attention reserved for common sources of misconception. And that appears to be the crux of the matter—in the past decade we have seen the results in school of misinterpretation or misapplication (inclusive "or") of the foundations of arithmetic. What implications does this have for the preservice education of teachers? Whether or not you will like this text depends largely on your philosophy in this matter.

The Mathematics of the Elementary School. Edward G. Begle. 1975, x + 453 pp., $12.00. McGraw-Hill Book Company, 330 W. 42 Street, New York, N.Y. 10036.

The unique feature of this book is the organization—topics in arithmetic, geometry, and probability are interspersed, but with care. This generally agrees with the use of mathematics in everyday life, where, for a nonroutine problem, the solution may involve counting, measurement, and computation, but seldom a single process.

The early introduction of geometry (chapter 3) saves the discussion of sets from its usual sterility. As a matter of fact, the concept of a set as a single entity distinct from its membership is difficult to convey with

symbols (curly brackets, commas, and so on); in contrast, the conventional sets of geometry—curves and regions—are traditionally viewed as single entities with no attention to their membership. Thus the juxtaposition of these topics gives a more complete characterization of the term *set*. Concomitantly, set operations become something more than games with marks on paper.

The informality of the treatment makes the caution offered in the preface a necessity—the book must be read carefully. The significant parts of any discussion are singled out, but can be missed if the reading is superficial.

The necessity for careful reading may be considered a drawback by some people; the book is also quite lengthy. A few inconsistencies show up—there is a difference between an exercise used to motivate a succeeding topic, and an exercise that assumes a fact not in evidence. A few instances of the latter are found in the problems at the ends of chapters. Traditional use of the word *intersect* (in geometry) to mean "nonempty intersection" is made without comment, and is especially disturbing in an otherwise careful discussion.

Topics in Contemporary Mathematics. Jack R. Britton and Ignacio Bello. 1975, xv + 487 pp., $13.95. Harper & Row, 49 E. 33 Street, New York, N. Y. 10016.

The strong feature of this book is the exercises. These are plentiful, interesting, and challenging; they also incorporate many applications from contemporary life.

Unfortunately, the expository portions are quite disappointing. There is, for example, a lengthy introductory chapter (chapter 2, 38 pages) on logic, yet in succeeding sections proofs and definitions are given without proper foundation. The proof that for integers, $a \cdot b = 0 \Rightarrow a = 0$ or $b = 0$ (p. 139) requires division of both sides of an equation by b, but division of integers is not defined, so the significance of the assumption that $b \neq 0$ is lost. For the student who never understood proofs in mathematics, this cannot be enlightening. Again, subtraction of integers on the number line is defined to be the opposite of addition (p. 141), but addition of integers on the number line is not discussed.

Distinctions such as those between number and numeral are often claimed to be antics with semantics; however, the importance of distinguishing between a physical situation, its mathematical representation, and the symbolism used to express it is exemplified in the following quotation (p. 144):

> Thus, with the integers alone, a famous Betty Cracker recipe calling for 3 cups of flour could not be cut in half, because $3 \div 2$ is not an integer. Likewise, you could not share a piece of candy equally with a friend, because there is no integer to represent $1 \div 2$.

Real life supplies the necessary counterexamples to these assertions.

Unified Mathematics: Content, Methods, Materials for Elementary School Teachers. Arnold L. Fass and Claire M. Newman. 1975, xii + 465 pp., D. C. Heath & Company, 125 Spring Street, Lexington, Mass. 02173.

As indicated in the title, this is intended as a multipurpose text, but the content loading seems sufficient to warrant its inclusion here.

The major premise for the content is (p. 38) that mathematics is essentially a language. As a consequence, the learning of mathematics becomes a kind of information processing, for one learns another language by mimicry or by learning its rules and how to apply them.

There are several drawbacks to the dissemination of information approach to the learning of mathematics. One is that exemplification must often be contrived. To illustrate that $6 \times (8 \times 4) = (6 \times 8) \times 4$ with a pile of blocks, for example (p. 102), the count must proceed from front to back in one case, from right to left in the other. The fact that this model cannot be made to work without appeal to the commutative property for multiplication is obscured, sacrificed to the aim of *making* it work. It would seem more instructive for the student to perceive the inherent difficulty with the model, since that would contribute to his understanding of spatial relationships. In contrast, the associative property for multiplication is not such a difficult idea that its verification should be contrived.

Also, there is always the danger of giving misinformation. An instance is that of stating (p. 17) that for sets, "the braces enclose the elements that are separated by commas." For the ensuing example it is tempting to visualize the Great Lakes between braces, with a giant comma being dragged to Buffalo (Niagara Falls?) to separate Lake Erie from Lake Ontario.

Out of the main stream . . .

Consciousness and Creativity: Transcending Science, Humanities, and the Arts. Bill Romney. 1975, 278 pp., $5.00. Ash Lad Press, P.O. Box 396, Canton, N.Y. 13617.

An interesting book, for reader reaction is not likely to be moderate.

Writing with the zeal of a new convert, the author manages to involve the reader in examining his own position on a number of critical issues—from dealing with a daydreaming student to the assignment of marks and the role of the teacher.

The current press for accountability makes it almost mandatory that educators confront their value systems. Some may prefer the confrontation to take place on a mental level, with the pros and cons neatly listed. But divested of all emotional loading, the resolution of an issue becomes a *policy*, not a *value*. The highly personal style in which this book is written may not appeal to all tastes; the book does, however, deal with educational *values*, not policies.

Metric materials

Edited by **Richard J. Shumway,** *The Ohio State University, Columbus, Ohio*

Code Used in Reviews of Metric Materials	
A—For adults	S—For students
T—For teachers	*—Good
L—For leadership personnel	?—Questionable

Metric Activity Cards A, B, and C. J. Norman C. Sharp and Elisabeth Hallamore. 1975, $33.00. Addison-Wesley Publishing Company, 2725 Sand Hill Road, Menlo Park, CA 94025. S

Set A consists of 32 cards on length, set B consists of 16 cards on area and 8 cards on temperature, and set C consists of 8 cards on volume, 7 cards on capacity, and 7 cards on mass. Each card is multicolored and 21 cm-by-30 cm in size. Each set of cards (A, B, or C) is accompained by a teacher's guide, which gives the objective, materials, procedure, and enrichment for each card. The materials seem best suited for intermediate or junior high students. The illustrations are colorful and attractive. The materials encourage estimation, involve measuring familiar objects, and do not require the purchasing of special equipment.—R.J.S.

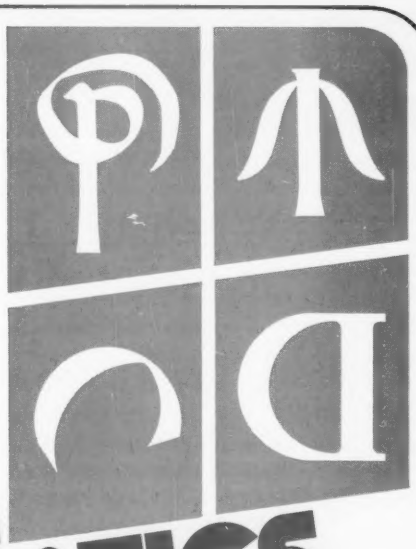
Metric Delights, a cassette/worksheet program on the metric system. 1975, $84.95. Creative Visuals, Big Spring, Tex. 79720. S, ?

There are twelve cassette tapes, one copy each of the corresponding twelve worksheets, and three sheets of grids and rulers. Topics covered include length, area, mass, and capacity. Strengths of the materials are that children are encouraged to do their own measuring activities during the lesson and no special equipment comes with the kit or needs to be purchased.

The tape on capacity titled "The Florist Who Could Never Make Up His Mind" illustrates some of the weaknesses of the materials. First, there is a story line involving two children making and selling clay figures, which has absolutely nothing to do with capacity or the learning activities but apparently is inserted periodically on the tape to give students a break. Second, it is almost impossible to detect the objective of the lesson. There is no statement of the objective and a careful study of the lesson suggests the following possibilities: introduction to the fractions 1/2, 1/3, 1/5, 1/8; or the multiples 2, 3, 5, 8; or the inverse relationship; or the ordering of the fractions; or estimating volume by using the height of the container; or, etc. It seems most inappropriate to let containers representing fractional parts such as 1/3 or 1/8 become the major part of a lesson about a system that essentially eliminates fractions in favor of decimals. Third, the tape contains coaching, which will certainly help students complete the activity, but it also reduces the learning involved. Fourth, the voices are sometimes fresh but other times sound like monotone reading. Finally, no student is likely to complete the lesson in less than 30 minutes; yet without a cassette tape the activity might take 10 minutes. In general, the tapes are extremely long, contain too much irrelevant material, and could be replaced by much briefer classroom activities.—R.J.S.

Metric Height Record. $3.95. Instructor Curriculum Materials, Instructor Park, Dansville, N. Y. 14437.S

The metric height record consists of a photograph of a rocket with centimeter markings from 0 to 192 together with a chart for recording each child's height. We put the chart on the classroom door in a third-grade class and found it to be most popular and used every day. The idea is great and most effective. Given the cost, many teachers may prefer to devise their own door pictures with centimeter markings and recording chart.—*R.J.S., and Gunta Nies, Burbank Elementary, Upper Arlington, Ohio.*



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1866

When the United States accepted the metric system

ROBERT G. CLASON

A professor of mathematics at Central Michigan University in Mt. Pleasant, Robert Clason also teaches classes for preservice teachers. He is a collector of old arithmetic texts, a hobby that grew out of his graduate work with Professor Phillip S. Jones. He is coeditor of Readings in the History of Mathematics Education, *an NCTM publication.*

The introduction of the Metric System into this country had been long recommended by scientific men, and by such statesmen as Madison, Jefferson, John Quincy Adams, etc. In 1866, through the influence of Charles Sumner, Congress authorized its use in the United States. . . . The system will, without doubt, in a few years be in general use in this country.

Edward Brooks, *New Written Arithmetic*, 1877

In 1866 a law was passed by the Congress of the United States and signed into law by President Andrew Johnson that made it "lawful throughout the United States of America to employ the Weights and Measures of the Metric System."[1] In support of the bill, Senator Sumner of Massachusetts expressed the task of the schools and their outcome as follows:

> [The metric system] must be taught in schools. Our arithmetic must explain it. They who have already passed a certain period of life may not adopt it; but the rising generation will embrace it and ever afterwards number it among the choicest possessions of an advanced civilization.[2]

With the passage of this law, some leaders expected the metric system to gain acceptance as the primary system of measures in the country, with the schools serving as a primary agency in the transition. Insight into the classroom effect of the 1866 law can be gained by looking at the arithmetic texts of the era. Examining the century-old attempt to "go metric" can offer a historical lesson for our current effort as well as supplying information for teachers who may soon find themselves in the role of experts, expected to be knowledgeable concerning the nature and background of the metric system.[3]

1. Quoted from Horatio N. Robinson, *The Progressive Higher Arithmetic for Schools, Academies, and the Mercantile Colleges Combining the Analytic and Synthetic Methods and Forming a Complete Treatise on Arithmetical Science and its Commercial and Business Applications*, (New York: Ivison, Blakeman, Taylor & Co., 1872), p. 431.

2. U.S. Department of Commerce, National Bureau of Standards, *U.S. Metric Study Interim Report: A History of the Metric System Controversy in the United States*, NBS SP 345-10, August 1971, p. 46.

3. Much of the information of this article stems from an examination of 49 arithmetic textbooks that appeared between 1800 and 1900 selected primarily by availability. I wish to thank Ms. Jan Pavlik and Ms. Cheryl Showerman for their able assistance with the text study.

PRE-1866 TREATMENT OF MEASURES

Following the American Revolution, treatment of measures in arithmetic texts was extensive by modern standards. This reflected the largely commercial purpose for teaching arithmetic and the variation in standards from place to place. For example, the units of cloth measure in the 1788 edition of Pike's *Arithmetic* included the nail, quarter, yard, ell-Flemish, ell-English, ell-French, and ell-Scotch. The commercial purpose is evident in the footnote:

> All Scotch and Irish linens are bought by the English or American yard, which is the same, and all Dutch linens by the Ell-Flemish; but are all sold in America by the American yard; though the Dutch linens are sold in England by the Ell-English, and the Scotch and Irish linens, as in America.[4]

From the Revolutionary era until after the mid-nineteenth century, units treated in textbooks evolved as might be expected in a period of minimal governmental action to establish a national system of measurements. Improvements were made, but they were partial and slow in coming. The confusion of three sets of common capacity measures, dry measure, wine measure, and beer measure (which, curiously, was used to measure milk[5]), evolved into the current confusion of two, dry measure and fluid measure (which, curiously, we use to measure flour). Firkins, kilderkins, puncheons, and butts, as well as various other units, became obsolete and were omitted from texts. But generally speaking, there was little effort made at systematic reform in treatment of the customary system.

Prior to 1866, treatment of the metric system in the arithmetics of the United States was rare. The single text examined that treated the system was Emerson's 1839 edition of *North American Arithmetic*, part of which is shown in figure 1. Emerson's section is noteworthy because it predated general use of the metric system even in France.[6] Emerson's respect for the metric system can be seen in his devoting almost three pages to it, as compared with slightly more than four pages to units of measure from twenty-seven other countries and regions. But his recognition of the metric system, as noted, was unusual. It wasn't until 1866 that any substantial treatment of the system was offered.

THE METRIC SYSTEM AFTER 1866

If inclusion of the metric system in texts before 1866 was rare, inclusion in texts after 1866, for grammar grades at least, was nearly universal. A common pattern was to add a metric section to the edition of a text appearing after the law was passed. For example, Thomson's 1853 *Practical Arithmetic* had no metric work; the largely identical edition of 1874 contains a metric appendix. Ray's *Practical Arithmetic* of 1857 does not mention the metric system; a revised edition, (the "one thousandth edition—improved," of this popular text) added a section on the "metrical" system that mentions the 1866 law. The 1866 version of the well-known mathematics educator Charles Davies' *Practical Arithmetic* contains no metric section (a matter of unfortunate timing?); the final section of the 1871 version is devoted to metric units.[7] These added metric sections tended to be self-contained, placed at either the end of the book or inserted at the end of

4. Nicolas Pike, *A New and Complete System of Arithmetic Composed for the Use of the Citizens of the United States*, (Newbury-Port: John Mycall, 1788), p. 44.

5. Of twelve texts dated between 1848 and 1870, nine treated both wine and beer measure. All nine mentioned that milk was sold using beer measure. Following 1870, beer measure faded from the texts.

6. While the metric system was made official in France in 1795, its use was not enforced. In fact in 1812 a transitional "usuelle" system was permitted for common use. The "usuelle" system included, for example, a redefined *livre* (pound) of one-half kilogram. Enough confusion resulted that in 1837 the French government took action making the metric system effectively mandatory beginning on January 1, 1841.

7. Rather surprisingly, Davies was a critic of the metric system. He chaired the committee whose 1871 report for the University Convocation of the State of New York stands as one of the few antimetric reports ever to come from an educationally-minded group. For more detail, see U.S. Department of Commerce, p. 51 ff.

FOREIGN WEIGHTS AND MEASURES.

The weights and measures of **GREAT BRITAIN** are the same as those of the United States, excepting the variations which are noted in the tables of ' Weights and Measures,' page 27.

The weights and measures of **FRANCE** being more nicely adjusted than those of any other country, will be here given the more fully on that account. It is, however, to be observed, that these weights and measures are according to a *new system*, not yet in very common use.

The fundamental standard adopted in France for the metrical system of weights and measures, is a quadrant of the meridian; that is to say, the distance from the equator to the north pole. This quadrant is divided into ten millions of equal parts, and one of these equal parts is called the METRE, which is adopted as the unit of length, and from which by decimal multiplication and division all other measures are derived.

In order to express the decimal proportions, the following vocabulary of names has been adopted.

For multipliers,

the word	*Deca*	prefixed, means	10 times.
"	*Hecto*	" "	100 times.
"	*Chilo*	" "	1000 times.
"	*Myria*	" "	10000 times

For divisors,

the word	*Deci*	prefixed, expresses the	10th part.
"	*Centi*	" "	100th part.
"	*Milli*	" "	1000th part.

It may assist the memory to observe that the terms for multiplying are Greek, and those for dividing, Latin

Thus, *Deca-metre* means 10 Metres.
Deci-metre " the 10th part of a Metre.
Hecto-metre " 100 Metres.
Centi-metre " the 100th part of a Metre; &c

Fig. 1. Page 250 in *The North American Arithmetic, Part Third for Advanced Scholars* by Frederick Emerson (Boston: Jenks and Palmer, 1839).

the chapter on denominate numbers. The first page of a typical section is shown in figure 2. A few texts treated metric units with customary units by types, meters with feet, liters with quarts, and so on.

Units of measure, of course, occur in the problems of many sections of textbooks besides in chapters devoted to denominate numbers. In a text seriously designed to promulgate the metric system, one might expect to see increasing use of metric units in such exercises. However, with few exceptions, this was not the case following 1866. Metric units were not integrated into other problem sets. Investigation of the first problem set using simple proportion in twenty-one grammar-grade texts of 1866–1900 revealed that only two used any metric units, although problems with customary units were common.

THE METRIC SYSTEM.

372. THE METRIC SYSTEM of Weights and Measures is based upon the decimal system of notation.

It was authorized to be used in the United States, and its use introduced into some departments of public service, in 1866, by an Act of Congress.

373. The *Principal Units*, or those from which the others are derived, are: —

The METER, which was intended to be, and is nearly, one ten-millionth of the distance on the earth's surface from the equator to the pole.

The SQUARE METER, which is the square whose side is one meter; and the ARE (pronounced *air*), which is the square whose side is ten meters.

The CUBIC METER, or STERE (pronounced *stair*), which is the cube whose edge is one meter; and the LITER (pronounced leeter), which is one thousandth of a cubic meter.

The GRAM, which is the weight, in vacuum, of a cubic centimeter of distilled water, at the greatest density.

374. The HIGHER DENOMINATIONS are expressed by prefixing to the name of the principal unit,

DEKA, 10; HECTO, 100; KILO, 1000; MYRIA, 10,000;

and the LOWER DENOMINATIONS by prefixing

DECI, 10th; CENTI, 100th; MILLI, 1000th.

NOTE. Kilo is pronounced *kil'o*, and Deci, *dēs'ē*.

375. MEASURES OF LENGTH.

10 *mil'limeters* (mm.)	= 1 *cen'timeter*,	cm.,	= .3937 inch.
10 centimeters,	1 dec'imeter,	dcm.,	3.937 inches.
10 decimeters,	1 METER,	me.,	39.37 inches.
10 meters,	1 dek'ameter,	dkm.,	393.7 inches.
10 dekameters,	1 hec'tometer,	hm.,	328 ft. 1 in.
10 hectometers,	1 *kil'ometer*,	km.,	3280 ft. 10 in.
10 kilometers,	1 myr'iameter,	mym.,	6.2137 miles.

Fig. 2. Page 325 in *Introduction to the National Arithmetic on the Inductive System Combining the Analytic and Synthetic Methods, in which the Principles of the Science are Fully Explained and Illustrated* by Benjamin Greenleaf (Boston: Robert S. Davis & Co., 1870).

THE METRIC SYSTEM IN HIGHER ARITHMETICS AND PRIMARY ARITHMETICS

A series of arithmetic texts of the 1866 era commonly contained several books besides the grammar grade texts just discussed. Often a text titled "Higher Arithmetic" was offered for high schools and normal schools (teacher preparation) students. Higher arithmetics normally followed the grammar grade pattern in adding metric sections following the 1866 law with the variation that the information included was often more detailed. For example, the metric section of Robinson's 1872 *Higher Arithmetic* ran 28 pages. It included a history of the metric adoption in France, the text of the 1866 law and its accompanying Congressional

resolutions (see fig. 3 for the 1877 version), a description of the two different surveys that had been made to obtain a standard meter from the definition of the meter as one ten-millionth of a quadrant of the earth's meridian, and even a set of metric units for measuring angles. But in spite of these extensive sections, the pattern of giving metric units minimal play in other sections was maintained in the higher arithmetics.

At the other end of the curriculum, series of arithmetics of 100 years ago commonly included one or more early-grade texts. These included units of measure both incidentally, at the primary level, and more formally usually in sections later in the book (see figure 4 for an example). What of

THE METRIC SYSTEM

OF WEIGHTS AND MEASURES.

797. The *Metric System* was adopted in France in 1795; its use was authorized in Great Britain in 1864; and in 1866, Congress authorized the Metric System to be used in the United States by passing the following bills:

AN ACT TO AUTHORIZE THE USE OF THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That from and after the passage of this Act, it shall be lawful throughout the United States of America to employ the Weights and Measures of the Metric System; and no contract or dealing, or pleading in any court, shall be deemed invalid, or liable to objection, because the weights or measures expressed or referred to therein are weights or measures of the Metric System.

SECTION 2. *And be it further enacted*, That the tables in the schedule hereto annexed shall be recognized in the construction of contracts, and in all legal proceedings, as establishing, in terms of the weights and measures now in use in the United States, the equivalents of the weights and measures expressed therein in terms of the Metric System; and said tables may be lawfully used for computing, determing, and expressing, in customary weights and measures, the weights and measures of the Metric System.

798. The **Metric System** of weights and measures is based upon the *decimal scale*.

799. The **Meter** is the *base* of the system, and is the *one ten-millionth* part of the distance on the earth's surface from the equator to either pole, or 39.37079 inches.

800. From the *Meter* are made the *Are* (air), the *Stere* (stair), the *Liter* (leeter), and the *Gram;* these constitute the *primary* or *principal* units of the system, from which all the others are derived.

801. The **Multiple Units**, or higher denominations, are named by prefixing to the name of the *primary* units the Greek numerals, *Deka* (10), *Hecto* (100), *Kilo* (1000), and *Myra* (10000).

Fig. 3. Page 453 in *The Progressive Higher Arithmetic for Schools, Academies, and Mercantile Colleges, Forming a Complete Treatise on Arithmetical Science, and Its Commercial and Business Applications* by Horatio N. Robinson (New York: Ivison, Blakeman, Taylor & Co., 1877).

13. How many quarts in 10 bushels?
14. How many pints in 3 pecks?
15. How many bushels in 64 quarts?
16. What part of a quart is 1 pint? 3 pints?
17. What part of a peck is 1 quart? 3 quarts?
18. What part of a bushel is 1 peck? 2 pecks?
19. A man sold 1 bu. 3 pk. of clover-seed at 8 cents a quart: how much did he receive?
20. A fruit-dealer paid $7 for 3 bu. 3 pk. of peaches, and sold them at 60 cents a peck: what was his gain?
21. How many pecks of chestnuts can be bought for $15.60, at 40 cents a peck? How many bushels?

LIQUID MEASURE.

Liquid Measure is used in measuring liquids; as, oil, milk, alcohol, etc.

The denominations are *gills*, *pints*, *quarts*, and *gallons*.

TABLE.

4 gills (*gi.*)	are 1 pint	*pt.*
2 pints	are 1 quart	*qt.*
4 quarts	are 1 gallon	*gal.*

1 gal. 4 qt. = 8 pt. = 32 gi.

Fig. 4. Page 140 in *First Book of Arithmetic for Pupils Uniting Oral and Written Exercises* by Emerson E. White (New York: American Book Co., 1890).

the metric system? Of nine of these first-level books only one contained a section on the metric system, and a brief inspection of this text failed to turn up any incidental use of metric units. It seems reasonable to conclude that the metric system was an upper-grade topic, not taught with early measurement ideas.

CONCLUSION

Using 20/20 hindsight, if the educators of 1866 truly expected to accept a major role in seeing the metric system assimilated into American life, there were at least two substantial weaknesses in the efforts they made. These were failure to offer "equal

time" to metric units in problems other than those of special sections devoted to the metric system and failure to expose children to metric units as soon as they were ready for measurement ideas. There are reasons for these shortcomings beyond error in judgment of authors and teachers, including antimetric sentiment that emerged in the years following 1866.[8] But whatever the reason, the result was that the metric system was the "second language" of measurement, the language less certain in a student's mind and hence the one he would prefer to avoid. Only when *liter* and *milliliter* have been given as many repetitions as *teaspoon, tablespoon, fluid ounce* (not to be confused with *ounce*), *fluid pint, fluid quart,* as well as the slightly larger *dry pint* and *dry quart,* will we as teachers be able to say that we have given the metric system a fair chance.

8. U.S. Department of Commerce, p. 82 ff.

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GCF and LCM on a geoboard

NEIL J. JEFFREY

Currently a mathematics teacher at York Mills Collegiate in Don Mills, Ontario, Neil Jeffrey wrote this article when he was a student in the Faculty of Education at Queen's University.

The theme that physical devices illustrating mathematical concepts enhance children's understanding of those concepts is an old one. Attempting to integrate various branches of mathematics is a current trend in mathematics education. For these reasons, I was particularly interested in the somewhat unexpected appearance on my geoboard of the greatest common factor (GCF) of the two numbers with which I was working.

I was considering the lattice of points in the Euclidean plane, each of which has two integral coordinates, and using a geoboard as a model of a part of this lattice. The greatest common factor appears in the following way. Given two positive integers, m and n, choose any point as origin. From the origin, mark the lattice point m units to the right, and from there, mark the lattice point n units up. Thus, three vertices of a right triangle have been determined. Now consider the hypotenuse of this triangle. If its two endpoints are the only lattice points contained in it, then clearly, it is separated by the lattice points into only one part. (Fig. 1) If there are other lattice points contained in the hypotenuse, it is not difficult to show that they separate it into parts of equal length. If there are $k + 1$ lattice points (including the vertices) contained in the hypotenuse, then it is divided into k equal parts. (Figs. 2 and 3) It is easy to see that the projections of each of these lattice points onto the other two sides (legs) of the right triangle fall on other lattice points, which separate each of the legs of the triangle into k parts of equal length. This means that k is a factor of both m and n.

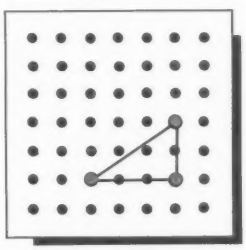
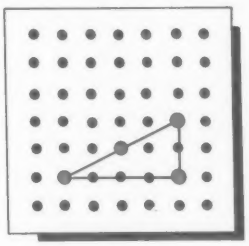


Fig. 1. GCF (3, 2)



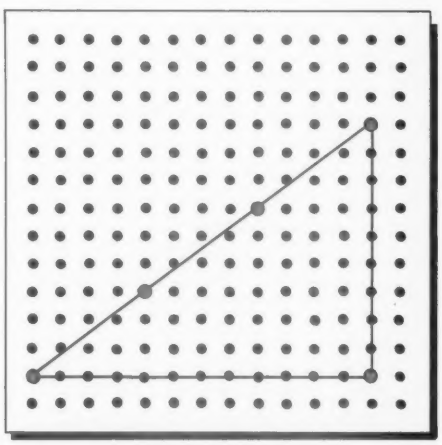


Fig. 3. GCF (12, 9)

Is there another factor of m and n greater than k? Suppose there is a factor of both m and n and it is greater than k; call it h. Then there are lattice points on each leg of the triangle that separate each leg into h parts of equal length. This implies that there are lattice points contained in the hypotenuse that separate it into h parts of equal length, which means that there are $h + 1$ lattice points contained in the hypotenuse. This is a contradiction since there are only $k + 1$ lattice points contained in the hypotenuse.

The proof that the greatest common factor of m and n is the number of parts into which the hypotenuse of the triangle is separated by lattice points is very similar.

Thus one can give students a very nice concrete representation of the notion of greatest common factor: make a right triangle with legs of lengths m and n. The greatest common factor of m and n [GCF (m, n)] is just the number of parts into which lattice points separate the hypotenuse.

Furthermore, the geoboard can be used to find the least common multiple of two numbers, m and n [LCM(m, n)]. Choose the origin at any point of the lattice. Mark the lattice points m, $2m$, $3m$, . . . units to the right of the origin and those n, $2n$, $3n$, . . . units up from the origin. This time, consider the isosceles right triangles with their legs pointing to the right and up from the origin (fig. 4). Find the smallest one that

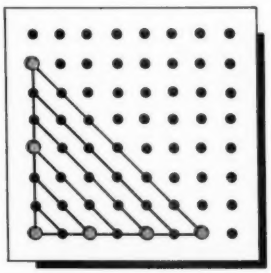


Fig. 4. LCM (2, 3)

has base angles located at previously marked lattice points. The length of the horizontal leg of this triangle is a multiple of m; the length of the vertical leg is a multiple of n. Since the legs are equal in length, that length is a common multiple of m and n. Since this is the smallest such triangle, that length is the least common multiple of m and n. Figures 5 and 6 show other examples.

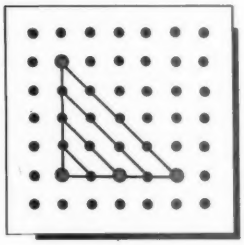


Fig. 5. LCM (2, 4)

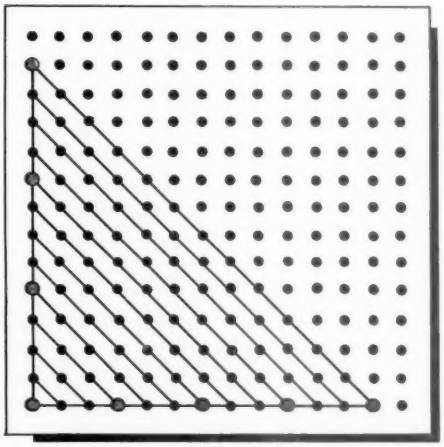


Fig. 6. LCM (3, 4)

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Arithmetic: must it be mechanical monotony and individualized boredom?

WILLIAM B. MOODY

As a professor of mathematics and education at the University of Delaware, William Moody teaches mathematics methods and content in the elementary education program. He has conducted research investigations into how children learn mathematics and is involved in arithmetic curriculum revision projects.

The recent movement in the direction of individualization of elementary school arithmetic curricula may well be leading many youngsters down a dull, seemingly endless road paved with computation problem after computation problem. Ever since the introduction of programs such as the *Individualized Prescribed Instructional* materials produced by the Learning, Research, and Development Center at the University of Pittsburg (along with other commercial and teacher-made versions of a student tailored curriculum), school personnel have been attempting to individualize their programs in arithmetic. It is time to take a close look at what is happening in the classrooms because, from what I have observed, the situation may be turning off more children than we realize.

One typical program of individualization employs the use of a leading publisher's textbook series for grades one through six. The company supplies the school with a set of level sheets numbered in sequence, and the teacher is instructed to test the students to determine the level to which they should be assigned. The youngsters are then given level sheets listing the textbook pages that are to be completed in a dictated order.

Let's assume that the child is at level twenty-two, which involves multiplication instruction and repetition. The company has gone through the books and listed all of the pages concerned with multiplication; the youngster is served a dose of the topic about as pleasant as three tablespoons of castor oil. I have witnessed children doing exercise after exercise based on one of the four computational processes. While this is going on, the teacher is busy grading the reams of papers these little clerks are turning out.

Since everyone is busy working and each child has a beautiful folder stuffed with evidence of his industry to present to mom and dad, no one questions the value of this individualized activity. The various agencies such as the state departments of public instruction and local district supervisors reinforce this activity by administering expensive standardized tests to children to rate and compare their performances to peers, districtwide, statewide, and nationwide. In this respect, the deluge of pages of computational exercises may help Mary answer more test exercises correctly, since it is easy to conclude that the best way to increase a student's probability of scoring highly on these instruments is to practice the type of exercises that appear on them.

I was talking recently to a sixth grader in a school that used a mathematics series that is color coded to enable students to work at

individual levels in the books by progressing from one color to another. When I asked him what he was doing in arithmetic, he said, division. A look at his folder revealed pages of division examples each containing ten or fifteen mechanical exercises. I was interested to learn that his chief reasons for working along on these boring exercises was to progress to the next color book and to help pass time in class so he could get outside and play. He pointed out that each year in school he had been directed to go through the same sequence of presentations of arithmetic: addition, subtraction, and multiplication followed by division. When I asked him if he was good at it, he said, "Well, I forgot a lot during the summer, but after I do a *couple* of problems I remember again."

During one of these individualized sessions, while the children were busy doing their division exercises, I asked some of them questions about the work. It was obvious that most of them had no idea of what they were doing. Their answers, which appeared in the form "209r2" meant nothing more to them than a written response to a mechanical exercise would be expected to mean. In this particular class, hours of presenting division as repeated subtractions and "stacking" the partial quotients on the right side of the example in the lower grades had given way to orders by the new teacher that everyone should be using the "adult method" of doing division. This meant covering over part of the dividend and asking, "How many __goes into__," then putting the number right over the number in front of the covered portion of the dividend.

The following completed example appeared on one child's paper:

$$\begin{array}{r} 29\text{r}2 \\ 15\overline{)3137} \\ \underline{30} \\ 137 \\ \underline{135} \\ 2 \end{array}$$

When the child was asked how she knew the answer obtained was correct, she immediately began to rework the exercise, employing the same mechanical procedure she used at the outset. It seemed a shame that she was so tied to the algorithm that she did not think to use estimation as a means of determining whether or not the quotient was reasonable. If she had understood what the division process meant, then she might have realized her mistake by observing that 30 fifteens is 450, therefore 29 fifteens could not possibly be over 3100.

In contrast, I once observed a six-year-old child who, when presented with the problem of having twenty-four Fudgesicles to be shared by four people, went through the natural analysis of repeated subtractions to arrive at the answer of six for each person. His explanation sounded something like this: "Twenty-four is made up of two tens and four left over. Each ten has two fours with two left over. If you put the two twos left over together, then you have another four." By counting the fours produced by breaking up the twenty-four, he found that there were six fours.

This sounds much more complicated to describe than it was for the child to carry out, because he wanted to solve a problem and not just to fit a mechanical procedure to a situation. If, at the age of eight or nine, after being drilled on arithmetic manipulations, he were presented with the Fudgesicle problem in class, I believe that the probability would be high that he would sit and ponder over which mechanical process should be employed.

I believe that we must ask ourselves what is happening to our youngsters in arithmetic classrooms, and whether mechanical skill at computation is what we really want to be striving toward. Certainly, individualized programs that emphasize computational drill are easy to organize, supervise, and evaluate (in fact little more is needed to carry out such programs than technicians and babysitters), but should individualization be wasted on low-level trivia?

The immediate reactions to my calling

computational skills in mathematics low-level trivia will no doubt classify me as an uninformed educator. I believe, however, that unless a person knows when to employ a computational skill, and what is actually happening during the process, that the ability to perform the act is worthless. Current programs pay little attention to these aspects of arithmetic because of their preoccupation with the act itself. What disturbs me even more is that the lower the level a child is working at, the more he is drilled at tasks that he may never know when to use. Often, one finds mentally retarded or educable youngsters doing pages of simple computational drill exercises in arithmetic. Those responsible are blindly presenting these children with the same routine that they understand all elementary school children should be subjected to, with little or no attention paid to solving problems faced in everyday life. The age of inexpensive electronic calculators, computerized banks and offices, and other technological achievements make it no longer legitimate to tell students that they must learn to perform computational skills so that they can carry out their day-to-day tasks. Instead, the emphasis must be on *when* to push the button that says add or divide and whether or not the response given is reasonable. (For this, they need some computational skills.) And perhaps, having some idea of what is happening when the button is pushed should also be emphasized. If this change in emphasis necessitates a search for justifications for studying arithmetic, perhaps the only justification we will be able to give is that one *might* study it for the pure enjoyment and aesthetic value of solving problems and for "seeing" the interrelated structure of the subject. From a practical point of view, the chief benefit of studying arithmetic is to open the door to studying additional topics and to the studying of mathematics and its related topics.

Then, if these are the real reasons for studying arithmetic, it should be obvious that individualized boredom involving year after year of concentrated attention to the mechanical processes of adding, subtracting, multiplying, and dividing does nothing to serve these reasons for studying arithmetic. Interestingly enough, this view was shared and expressed by the members of the Cambridge Conference on the Correlation of Science and Mathematics, when, in their 1969 report, they made the following statement:

> The present report advocates that the student confront mathematics in the context of solving real problems. It may therefore be surprising that the majority at our conference recommended that the teaching of the standard long division algorithms not be required in the elementary school. A less formal teaching of the long multiplication algorithm was also suggested. Some of the conferrees would want a version of the algorithms to be taught in some manner before students reach senior high school. There was nearly complete agreement, however, on the following points:
>
> (1) A very large amount of school time is devoted to learning and practicing the algorithms. Typically, work on those for long multiplication and long division is started near the end of grade 3 and continues with ever increasing emphasis through grade 6. When any topic occupies a large amount of time, we must ask if it is really that important.

(The Cambridge Conference on the Correlation of Science and Mathematics in the Schools was a workshop-style conference held in 1967 under the support of the National Science Foundation. The participants were well-known scientists and educators in mathematics and science from across the nation. The conference centered its attention on making suggestions for improving the mathematics and science curriculum of the schools.)

What then are the alternatives? Introducing a curriculum filled with new topics and terminology such as set theory, numeration systems, and nonmetric geometry does not address the problem. Instead, we should strive for a curriculum that is designed to help children know when to add or divide, and that would emphasize many different ways of carrying out computational processes. The major ingredient of such a program should be problem solving with functionally related computational activities. This means that we must abandon the hypothesis that children must master the computational skills before they are capable of considering problems involving these skills.

A thorough examination of typical textbook approaches to computational skills reveals just how prevalent this hypothesis is. Most begin with "simple" exercises employing one- or two-digit numbers and then progress to the more "difficult" two- or three-digit exercises. The sequences then normally end with a page or two of word problems calling for the student to apply what he has just mastered.

The end products of this entire procedure are students exhibiting a total lack of imagination in dealing with anything arithmetic: students who may be able to tell you how much 4×23 is if they have access to pencil and paper, but who cannot give you even a gross approximation without them.

As an alternative, students should be encouraged to attack problems in an informal fashion. They should be shown how to estimate their way along, and, by the same token, teachers should be satisfied with reasonable approximations as results. Then, when confronted with 4×23, a student could at least tell you that the product will be slightly less than a hundred, which is four twenty-fives (4×25). By the time he can tell you that 4×23 is the same as 4×25 minus four twos, the student will be well on his way toward developing the need for a more general computational algorithm. By being allowed to extensively explore the area, he will have found situations in which arrival at accurate estimates are quite difficult, and he will have found some situations in which an exact answer is desirable. He will have also developed background and facility in dealing with the multiplicative process that will give the algorithm some "meaning" instead of it just being an unthinking liturgy.

Let's consider a practical example of this generalized procedure for, say, the process for finding a certain percent of a number. In a fifth- or sixth-grade class a student would no doubt be presented with a formal procedure for finding 15% of \$16.80. It would probably include changing the percent to a decimal and then multiplying the numbers and placing the decimal point of the product four places in from the right of the numeral.

Is this the way one might attack the problem in an everyday encounter? I had the opportunity to observe my son and a friend, both entering seventh graders, attack this problem and was again made aware of how unrealistic our school-taught computational procedures are when put to a practical test. The boys had a friend selling golf balls for them since circumstances had made it impossible for them to appear on the course. They had agreed to pay the boy 15% of his total sales, which came to \$16.80. How much was the boy to get?

They obviously did not transfer the procedure they were exposed to in school to this situation, but instead resorted to some wild guesses and if left alone the young salesman would have profited from their ineptness, but dad could not let this opportunity pass without conducting an impromptu lesson on percent. When asked if they knew 10% of \$16.80 they hesitated and replied \$1.68. They were then asked if they could use this conclusion to arrive at 5% of \$16.80, to which they finally replied \$ .84. One of the boys instantly caught on and responded by stating they owed their salesman about \$2.50 for his efforts.

The above attack seemed like a very practical method of solving the problem and reminded me of the procedure used to arrive at my gas mileage when filling the tank of my automobile. While sitting be-

hind the wheel, it is impossible to divide 250 miles by the 16 gallons as presented in the school textbooks. So instead, I use an approximation technique and a compelling desire to arrive at an exact figure leads me to the correct answer. The thought process might include a pattern like the following: "Let's see if I am getting 20 miles per gallon. That would be 320 miles, so it is less than 20. Now 10 miles per gallon would result in 160 miles and half of that is 80 miles, so I must be getting about 15 miles per gallon because 160 and 80 are 240."

The arithmetic curriculum in the classroom usually does not encourage or even suggest that youngsters might attack problems in this intuitive fashion. The emphasis is always on the mechanical rule and is based on a false assumption that people will transfer the rule to practical situations. At present I am dismayed by the almost total lack of imagination exhibited by students when working arithmetic problems in school. Although there have been attempts to revise the arithmetic curriculum in the schools, more frequently labels such as "new math," "new, new math," "math stations," and "open classroom approaches" are merely camouflages for the continuation of the same old diet when it comes to what children are actually doing at their desks. Even the creation of completely new curriculum materials under these labels does not guarantee substantial change. Parents, teachers, students, and administrators must become aware of, and must also be willing to admit that a mass of time is wasted by students in arithmetic classrooms doing outdated computational routines. Then, and only then, can we begin to concentrate our efforts on giving arithmetic its rightful place as a thinking man's tool.

Reference

Cambridge Conference of Science and Mathematics in the Schools. *Goals for the Correlation of Elementary Science and Mathematics.* Newton, Mass.: Education Development Center, Inc., 1969.

See also:

The *Arithmetic Teacher*, November 1975, vol. 22, number 7.

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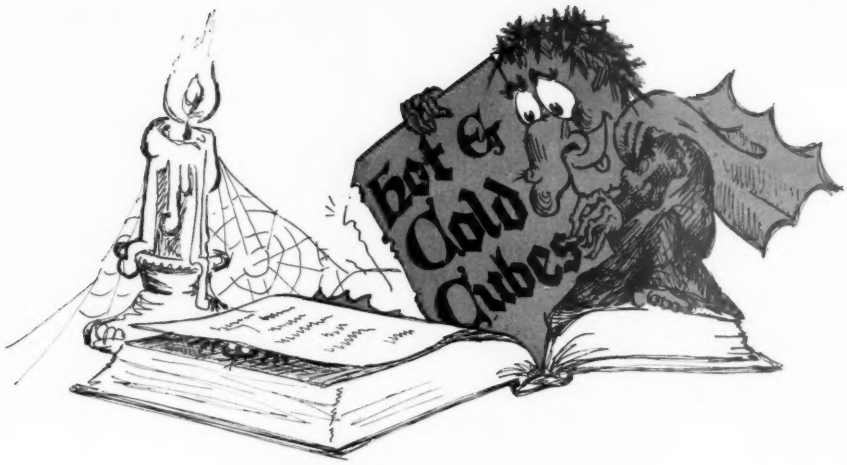

STANLEY M. JENCKS and
DONALD M. PECK

Working together in the preparation of elementary teachers in mathematics and its teaching, Stanley Jencks and Donald Peck also teach classes in methods for secondary school teachers of mathematics and conduct mathematics courses for remedial college students.

A simple model that portrays both addition and subtraction of positive and negative numbers is hard to find. A few years ago the authors ran across one that has been so successful it must be passed on for other teachers to use. This model lies in the realm of fantasy yet provides an easily imaginable and crystal-clear picture of how positive and negative numbers behave when they are added or subtracted.

Imagine a witch stirring up a caldron of brew. The caldron contains vast numbers of hot and cold cubes, which control the temperature of the brew. These cubes are weird things that only witches can make; the cold cubes are similar to ice cubes except they don't melt, and the hot cubes are something like charcoal briquets that don't lose their heat. If the same number of cold cubes as hot cubes are in the caldron the temperature of the brew is 0°. For each hot cube that is put in the temperature goes up one degree; for each one removed the temperatue goes down one degree. Similarly, each cold cube added lowers the temperature one degree and each cold cube removed raises it one degree.

Now let's get a problem going—suppose the temperature of the caldron is +4°. (A surplus of 4 hot cubes would do this.) Then 10 cold cubes are dumped in. What will the temperature of the brew be? (Since each cold cube put into the caldron lowers the temperature one degree, the addition of 10

cold cubes will lower it 10 degrees to −6°.) Thus,

$$^{+}4 + {}^{-}10 = {}^{-}6.$$

For another example, what would be the temperature if you started with brew at $^{+}3°$ and took out 2 cold cubes? (When a cold cube is removed the temperature goes up one degree. The removal of two cold cubes would make it rise two degrees to $^{+}5°$.) Hence,

$$^{+}3 - {}^{-}2 = {}^{+}5.$$

Notice that in this model each plus and minus symbol is used in a natural way. For example, in $^{-}3 - {}^{+}5 = {}^{-}8$:

(*a*) $^{-}3$ is the beginning temperature of the brew.

(*b*) The minus sign of operation signifies "take away." This is the natural way a beginner thinks about subtraction.

(*c*) $^{+}5$ denotes 5 hot cubes.

(*d*) $^{-}8$ denotes a temperature.

The association of the symbolism with the student's perception of temperature changes is what makes this model so useful to beginners.

If a teacher uses just a few examples in which students discuss and convince each other what ought to happen from the model, the problems of learning addition and (especially) subtraction are disposed of without difficulty. If at any time students become confused, they are asked to again visualize the model (rather than to recall a rule) and to describe what happens to the temperature of the brew.

Almost every student quickly learns that problems like $^{-}5 - {}^{-}2$ and $^{-}5 + {}^{+}2$ yield the same answers. After all, taking out 2 cold cubes raises the temperature 2°, and so does putting in 2 hot cubes. What could be simpler? Students themselves discover the broad generalization that subtracting a number and adding its opposite are really the same operation.

We have shown this model to a lot of teachers and we have yet to find one who has not found instant success with it. What some teachers have formerly taught as rules or as ideas that have required quite a bit of "structure" behind them become so simple that the rules and structure hardly seem necessary. As long as a student carries this simple and imaginable model in his mind he doesn't need any memorized rules!

Individualized teacher preparation programs for students and faculty through a CBTE model

ROBERT G. UNDERHILL

CBME (Competency Based Mathematics Education) has been a major interest of Bob Underhill's for several years. He chaired a committee on CBME for the Texas Education Agency. He has also developed an extensive self-paced media program of sound filmstrips for teachers.

A program of competency-based education was begun in the mathematics component of the elementary education program at the University of Houston in 1971. In the ensuing years, many changes have been made and the following major characteristics have emerged:

1. Competency-based
2. Highly individualized
3. Use of modules
4. A diagnostic model
5. Extensive use of media
6. Mathematics laboratory
7. Differentiated staffing
8. Strong clinical experiences component
9. Broad professional base
10. Affective concern

The program was the topic of a one-and-one-half day conference immediately preceding the annual meeting in Houston. Since that time many inquiries have been received.

A COMPETENCY-BASED, DIAGNOSTIC, INDIVIDUALIZED PROGRAM

At the beginning of the semester, each student purchases a 65-page booklet of modules (now in their seventh edition). The modules comprise a course map. They indicate the competencies to be mastered, the methods and schedules of evaluation, pretest items, and suggested enabling activities.

Competencies

Teachers must possess many types of skills and knowledges. Our program focuses on two types that differ primarily in how they are evaluated.

1. Knowledge competencies—evaluated primarily by paper and pencil. The person whose competence is being validated need not be present during evaluation.

2. Performance competencies—evaluated by observation procedures. The person being evaluated must be present.

Knowledge objectives consist of competencies demonstrated on tests or written work turned in to instructors. Performance objectives consist of competencies demonstrated in the mathematics laboratory at the university or in the public schools while the student is working with children.

Evaluation

Each knowledge objective has one or more informal pretest items in the module for self-assessment. Success on these items indicates strengths and weaknesses. Students

may choose to complete one or more enabling activities before attempting formal assessment. Since each objective has a criterion level of two out of three, or three out of four, students may be evaluated on six, ten, or even fifteen objectives at one time. A record is maintained of those mastered and the student is reevaluated only on those on which he has not met criterion level. Evaluation sessions are held about every two weeks for knowledge objectives. The program has approximately sixty knowledge objectives.

Each mathematics laboratory objective has a pictorial or semiconcrete evaluation in the pretest. In the mathematics laboratory, the student must demonstrate his understanding of and ability to teach with such diverse aids as Cuisenaire rods, abacus, graph paper, geometric models, fraction kits, and so on. Students can return for a second or third evaluation if they are not successful. In the interim, they can complete additional enablers and observe in the mathematics laboratory. The program has approximately twenty-five mathematics laboratory objectives.

Each public school performance objective had a pretest item, which necessarily closely approximates the form of the formal assessment. The way to see if one can teach a mathematics topic or concept concretely is to try it. These competencies are evaluated in the schools while the students are working with children. They, too, can be repeated if not mastered on the first try. The program has approximately twenty public school objectives.

Individualization

Each student receives a list of competencies to be demonstrated, pretest items, and a list of enablers for *each* objective. A student then decides which competencies to master within a broad framework of prerequisites. (For example, a student must meet knowledge level competencies on multiplication before he can *teach* multiplication.) Each student decides when to be evaluated on a given objective and whether to work by himself or with a group.

Media

In addition to the customary enablers of texts and other written materials, the College of Education Learning Resources Center (LRC) houses many mediated presentations for enablers. Eighteen slide-tape presentations consisting of over 650 slides and audiotaped lectures teach teachers how to teach mathematics. The information helps teachers learn the use of manipulatives, pictures, and diagrams.

Other mediated information includes minilectures with outlines and other accompanying printed matter on topics such as general diagnosis, clinical diagnosis, readiness, decision-making, affective concerns, and individualization. Many other resources are available through videotapes and games. All mediated materials are catalogued in the modules *by objective*. They are housed in the LRC, which is an open-stacks facility open to individuals and groups about 60 hours each week.

Affective needs

Students become aware of their strengths and weaknesses. They greatly appreciate opportunities to be reevaluated and continue to grow. Success is further insured by a careful guidance through diagnosis of two children. Instruction following diagnosis is nearly fool-proof and students gain much security and self-confidence through these experiences. Great emphasis is placed on helping *children* understand; the best by-product is the growth of teachers as *they* understand what they prepare to teach.

DIFFERENTIATED STAFFING

A major concern in individualizing learning and evaluation is the time element. Successful use of personnel who possess requisite skills is the answer. Two major breakthroughs have occurred.

1. *Undergraduates.* Students who successfully complete the course may sign up for one, two, or three semester hours of credit to work in the mathematics laboratory. The teaching experience, working

with peers, establishment of communications skills, and achievement of new insights have proven invaluable to individuals who have sought and fulfilled this experience. Students in the course find this to be nonthreatening, and there has been no experience of impropriety. Students identify strongly with these skills.

2. *Public school teachers.* Teachers whose children are tutored engage in an inservice program designed to acquaint the teachers with the program organization, objectives, and evaluation criteria.

Usually, there are about three university staff, four teachers, 120 children, and sixty college students in a school cafetorium at once. Each college student works with two children for 45 minutes each day of the school experience. This yields a supervision ratio of about 1 to 10. Each supervisor examines a card filled out by each student and determines whether the competency has been met. The card contains the student's name, the children's names, objective, materials, and competency.

In the fall of 1974 a plan designed to further meet the individual needs of staff members was implemented. Each task was assigned a number of hours, based on previous experience, and staff members could select the tasks in which they preferred to participate, such as test construction, supervision, mathematics laboratory conferencing, test grading, inservice, and so on.

SCHEDULE

The following schedule evolved from the experiences of six semesters:

Weeks, 1–5	On campus. Introduction and knowledge objectives
Weeks, 6–8	In schools. Performance with children, clinical diagnostic techniques, high interaction teaching.
Week, 9	On campus. Knowledge and preparation for schools.
Weeks, 10–13	In schools. Performance with children, class diagnostic techniques, low interaction teaching.
Weeks, 14–15	On campus. Knowledge and course synthesis.

The instructional model is diagnostic. In preparation for the first field experience, each student must satisfy knowledge and mathematics laboratory competencies in one content area. Then he performs a clinical evaluation and teaches in a high-interaction mode. Then he selects a second content area for the second field experience. This time he uses classroom diagnostic procedures of survey and analytic testing and teaches in a low-interaction mode.

PROGRAM SUMMARY

Mathematics teacher education programs that meet the needs of students *and* teachers can be created. Competencies are helpful in delineating goals, and modules and media are very useful tools in providing the instructional delivery system.

Four major goals can be realized through use of a system such as the one described.

1. A diagnostic model of teaching can be *experienced* by the learner and *practiced* by the learner.

2. A strong clinical experience component can be included.

3. Teacher training can become a job of the *profession.*

4. Affective needs can be accommodated.

CONTINUING CHALLENGES

Two major challenges remain on the horizon as this model continues to evolve.

Lock-step. The conventional one-semester course is a thorn. As the course currently exists, students can complete all objectives, *except* those in the public schools, as quickly or as slowly as they wish. The vast majority of students require exactly one semester. A few finish in thirteen weeks, and a few receive incompletes. The major problem to overcome is the rigidity of the school experience. A cycling of

school experiences every two or three weeks to extend to students more flexibility in the commencement and termination of those competency demonstrations is being tried.

Public school personnel. We are committed to a professional education model that includes university and public school personnel. However, we still have some difficulty communicating our goals and evaluation procedures. We wish to facilitate higher reliability and validity of teachers' decision-making during supervision. The answer is probably in the development of better evaluation criteria. We are pursuing this course of action.

Books and materials

Criterion Arithmetic Remediation and Enrichment System. Yvonne Foote, Donald Foote, and R. G. Heckelman. San Rafael, Calif.: Academic Therapy Publications, 1975.

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Math Chalk Talk. Joyce Gruenberger. Los Angeles: Zavell Math Company, 1974. 54 pp., $3.95.

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Measurement, 2–8, Level Orange and Level Blue. Alexander Tobin, Naomi Taylor, and Alan Barson. New York: W. H. Sadlier, 1976. $1.44; Teacher's Annotated Edition, $2.52.

100 Geometric Games. Pierre Berloquin. New York: Charles Scribner's Sons, 1976. $6.95.

Verses to Reckon With. Sidney R. Levine. New York: Hart Publishing Company, 1976. 192 pp.; $6.94 hardcover, $2.95 paperback.

* * * *

Bases Other Than Ten. 9 lessons. 9 cassettes, spirit master study guides for each lesson, spirit master placement exercises, spirit master practice exercises, teacher's guide, and spirit master posttest. New York: Miller-Brody Productions, Inc.

Beginning Addition and Subtraction Unit. Andrea Winters-Weston and Irma Hillman. 1975. $30.00. New York: Miller-Brody Productions, Inc.

Decimal Number Computation, 12 lessons. 12 cassettes, spirit master study guides for each lesson, spirit master placement exercises, spirit master practice exercises, teacher's guide, spirit master posttests, individual and class progress charts. 1975. $154.00. Miller-Brody Productions, Inc.

Fractional Number Computation, 15 lessons. 15 casetes, spirit master study guides for each lesson, spirit master placement exercises, spirit master practice exercises, teacher's guide, and spirit master posttests. 1976. $180.00. New York: Miller-Brody Productions, Inc.

Introduction to Fraction Concepts, 8 lessons. 8 cassettes, spirit master study guides for each lesson, spirit master placement exercises, spirit master practice exercises, teacher's guide, and spirit master posttests. 1975. $99.00. New York: Miller-Brody Productions, Inc.

Learning Readiness Unit. Andrea Winters-Weston and Irma Hillman. 1975. $50.00. New York: Miller-Brody Productions, Inc.

Math Facts Mastery Testing Pack; Addition-Subtraction. Mildred M. Dominy and Michael T. McGaulley. 1973. $65.00. New York: Miller-Brody Productions, Inc.

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An approach to teaching numeration systems

LINDA M. VERBEKE

A teacher at the Engleman School in Center Line, Michigan, Linda Verbeke is currently teaching seventh-grade mathematics, algebra, and geometry.

Of what use is the study of different numeration systems? There are at least two answers to this question: Numeration systems are interesting in themselves and important from the historical standpoint. The study of numeration systems gives students a better appreciation and understanding of the development of the decimal system, in much the same way that the study of a foreign language helps students to understand their mother tongue.

By the time they reach the seventh grade, students are expected to perform the four basic operations in base ten at the adult level. They are then likely to accept arithmetical computations as a well-established habit and it is therefore almost impossible to get students to examine closely the structure of the base ten numeration system. The study of early numeration systems, early computing devices, and other possible numeration systems is a way of developing an appreciation for the ingenuity of the decimal system.

The study of early numeration systems and other base numeration systems can be most rewarding if students can get actively involved. In my seventh-grade classes, students' understanding is enhanced by their building and demonstrating models and giving oral reports on their research findings. We approach the topic by looking at the historical development of numerals and counting. I usually lead into the topic by starting with the historical progression of man. We then look at a chart of early forms of present numeration systems, noticing differences and similarities (symbols, symbol for zero, place value, type of place value, number of basic symbols, limitations of system, and so on). The class then breaks up into small groups of two or three and each group chooses one of the following topics to work on.

1. Make a report on the development of our present symbols for numbers. Explain when and how they originated.

2. Collect the names of numbers in different languages. Note any similarities in the names and try to explain them.

3. Make a Russian abacus (s'choty) and demonstrate how to use it.

4. Make a Japanese abacus (soroban) and demonstrate how to use it.

5. Make a Chinese abacus (suan pan) and demonstrate how to use it.

6. Make a model of a Babylonian clay tablet.

7. Make a quipu such as the Incas of Peru used to keep count of their sheep.

8. Make a collection of counters—pebbles, sticks, tally sticks (a device of the Middle Ages), and notch sticks. Tell how each was used.

9. Make a set of Napier's bones and demonstrate how to use them.

10. Make a chart entitled "Numerals—Past and Present." Include Egyptian, Babylonian, early Roman, Mayan, Chinese, early Greek, Hindu, Arabic, Italian, and Spanish numerals. Note any similarities or differences.

In addition to creating interest, enhancing historical appreciation, and giving a little insight into the decimal system, these projects also make a great display that can be shared with other classes. The discussion that follows the presentations of the projects can lead into other informal, interesting topics—the scratch system, the doubling system, the lattice method of multiplication, Russian peasant multiplication, finger multiplication, Trachtenberg system of addition, multiplication in Roman numerals, and simple adding machines.

Most students are somewhat surprised to discover that the use of a base of ten is not mandatory. (We would probably have used a system based on groups of eight if man had had only eight fingers.) I introduce the idea of other numeration systems by asking the class to become an inventor of a new, unfamiliar system that has exactly the same structural characteristics as the base ten system. For example, we might examine base five and base eight. The study of the base five and base eight numeration systems leads to a generalized pattern for all numeration systems that are arranged according to the positional principle. If the base of the system is n, the place value can be represented in the following way:

n^4	n^3	n^2	n^1	n_0
□	□	□	□	□

After looking at place value, addition, subtraction, and multiplication in different bases, and when I feel the students have a fairly good understanding of the idea of nondecimal systems of numeration, they again divide up into small groups and this time choose one of the following topics to work on.

1. Write to the Duodecimal Society of America and report on the information you receive. What are some of the unusual features of base twelve?

2. Using the game rummy as a model, make a mathematical card game on numeration systems. (Ex. One book of four cards could be 110_{three}, XII, 22_{five}, and 14_{eight}.)

3. Make a mathematical bingo game for testing understanding of different numeration systems. Problems are used in place of call numbers. Make at least thirty different bingo cards out of poster board. (This project can then be used by the whole class for review purposes.)

4. From wood or poster board construct models for working in other bases. Make enough of each so that counting, addition, and subtraction can be done using the models. (See fig. 1.)

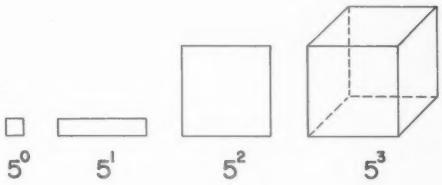

Fig. 1

5. Make an abacus for use in a base other than ten and be able to count, add, and subtract on it.

6. Make a slide rule for adding and subtracting in a base other than ten. Demonstrate how to use it.

7. Make a binary counter and tell how to use it. Where would a binary counter be used?

8. Make a calendar using different bases for each month.

9. Make and explore the game of Nim. How can base two be used in formulating a winning strategy?

10. Make models of Napier's bones out of a durable material such as heavy cardboard or wood. Make the bones for different bases to illustrate or check computations with numerals in bases other than ten.

11. Build nomographs for computations, such as addition, subtraction, multiplication, and division, in bases other than ten.

12. Explore addition, subtraction, multiplication, division, symbols, properties, and so on, in base n where $n \geq 2$.

To test the students' knowledge and understanding of material at the conclusion of this project I have them create their own numeration systems. Most students enjoy this activity so much they invent interesting little stories and cartoons to present their creations.

By presenting the topics of numeration systems in this way I find students have a better grasp of the ideas and the following objectives are accomplished:

1. Students gain further insight into the decimal system.

2. Students are provided a historical setting for the study of the Hindu-Arabic notation system and they consider significant aspects of this system.

3. Students consider the differences between numerals and numbers.

4. Students examine the general principles at work in a positional numeration system.

5. Students contrast different types of place value (base ten and Roman).

6. Students contrast nonpositional systems with positional systems.

7. Students gain an appreciation of the historical development of mathematics and its place in human society.

8. Students are actively involved.

The insights into the decimal system gained through the study of other numeration systems, the building of models and projects, and the creating of a numeration system are invaluable to our students, especially for work in the future when they may be using calculators and computers. And, too, students are curious, eager, having fun, and learning.

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Sometimes material leads to discovery

FRANCES B. MORGENSTERN and
MORRIS PINCUS

Respectively, principals of Public School 195 and Public School 194 in Brooklyn

The use of manipulative materials for the exploration and discovery of mathematical ideas or relationships, and for making symbols and operations meaningful, is considered an important feature of mathematics programs today. However, sometimes materials are used in too casual or transitory a manner. The fact that the material lends itself to discovery and to understanding does not automatically insure that discovery or understanding will occur.

Children need to have sufficient use of a material to become familiar with the structural properties of materials and to use them to find solutions to problems. The materials become conceptual models that can be visualized and referred to in the solution process.

Recently an incident occurred that illustrated this point. A fourth-grade teacher brought to the authors a cluttered and seemingly incomprehensible record of computations for the example $7\overline{)446}$. Somehow, she said, the boy had gotten the correct answer but he didn't seem to have an understanding of the process that she had taught. He was a pupil of average ability, she indicated, and she had reviewed the long division method several times. When she tried to get him to explain what he had done, he mentioned Dienes blocks, but not, it seemed, in any way related to division. The Dienes blocks and other materials were kept in a "math-lab corner," but they were used for the most part for enrichment, as for example, in the study of different number bases. When it came to a basic topic such as long division, the teacher carefully went through the procedure step by step. Then the children were given additional examples to do on their own. The teacher was familiar with some errors—pupils forgot one step or another in the sequence, or made careless mistakes. This paper, however, seemed to contain none of the common errors. Instead, there was a collection of multiplications by 7 and by 3 that didn't make much sense to the teacher but somehow led to the right answer.

We asked the child to tell how he solved the example $7\overline{)446}$ and here is what he said:

"I figured out there were 44 tens and 6 ones in 446. I took one tens Dienes block and I saw that it was a seven and a three. But that was only 1 ten. So in 44 tens, I have 44 sevens and 44 threes. So 44 is part of the answer. Then I timesed 3×44 and got 132. Add the 2 to the 6 ones from before because 132 has 13 tens and 2 ones. In the 13 tens you have 13 sevens and 13 threes. The 13 sevens is part of the answer. The 13 threes is 39. Add the 9 to the other ones. That makes 6 and 2 and 9 ones. The 3 tens is 3 sevens and 3 threes. The 3 sevens is part of the answer. The 3 threes is 9. Add the 9 to the ones. That gives you 60 sevens in the answer and 26 ones. The 26 ones is 3 sevens and 5 left over. So the answer is 63 sevens R 5."

When asked why he didn't pursue the same method when he had 26, he responded that he could have but this way

was easier. Omitting the verbiage, we reconstructed his method as shown in figure 1.

446 = 44 tens		+ 6 ones
44 tens	= 44 sevens + 44 threes	
	= 44 sevens + 132	
132 = 13 tens		+ 2 ones
13 tens	= 13 sevens + 13 threes	
	= 13 sevens + 39	
39 = 3 tens		+ 9 ones
3 tens	= 3 sevens + 3 threes	
	= 3 sevens	+ 9 ones
		26 ones
26 ones	= 3 sevens	+ 5 ones
	63 sevens	+ 5 ones

Answer is 63 R5

Fig. 1. Child's solution for $7\overline{)446}$

Our involvement in trying to help the teacher locate and correct the source of this child's "errors" turned out to be one of those situations in which the process of seeking information on one question leads to insight unexpectedly into another. Although the teacher had used materials to illustrate place value (4 hundreds, 4 tens, 6 ones) and had systematically developed the long division algorithm, somehow this boy had lost the connections among the steps in the division process. He had at his disposal the following components of experience and understanding:

1. The meaning of $7\overline{)446}$ as, "how many sevens are contained in 446?"

2. The meaning of 446 as 44 tens and 6 ones.

3. (And this was a surprising insight for the writers.) The boy had a perceptual grasp of the distributive property (44 tens is 44 sevens and 44 threes) presumably as a result of using the Dienes blocks to study different number bases. He used this visual model of each ten as a seven and a three as the basis for determining the number of sevens in 446.

The method is not an economical one in terms of time and work—but this may not be important, especially at the introductory stage. It is not an efficient procedure in view of the number of possible occasions for making careless errors, which, of course, is a good reason for refining the algorithm. It illustrates clearly, however, the manner in which flexibility and independence can be gained by a child who has had the opportunity to use a material in such a way that its properties are understood.

It is only at this point that the introduction of an algorithm is meaningful, when material retained as a conceptual model can be used as a reference from which information and relationships can be derived in many unanticipated ways. We are not advocating that original derivations of this nature be substituted for a ready familiarity with the multiplication and division facts, nor would we anticipate that all children would perform in this manner. In fact, it is likely and desirable that children will perform in their own individual fashions in the early stages of development and will develop their own ways of refining and recording the method used. In discussing and comparing the various algorithms developed, the advantages of the standard algorithm will become clearer and more meaningful. It is our feeling, however, that children who do use materials to discover structure and relationships develop a depth of understanding, an ability to operate with numbers, and a sense of satisfaction that augur well for their future as students.

An easy way to change repeating decimals to fractions—Nick's method

DENNIS ROBIDOUX and
NICHOLAS MONTEFUSCO

Western Hills Junior High School in Cranston, Rhode Island, was the school, Dennis Robidoux, the teacher, and Nicholas Montefusco, the student in the discovery described here.

During a pre-algebra class on changing repeating decimals to fractions, Nick noted a relationship between the original problem and the answer and proposed a method for finding the fraction. After checking this method through various types of problems, it appears that the method is feasible.

For example, change $.6\overline{3}$ to a fraction. First, let's look at the formal method of changing repeating decimals to fractions.

$x = 0.6\overline{3}$ Set x equal to the repeating decimal.

$10x = 6.3\overline{3}$ Since there is one digit repeating, multiply both terms of the equation by ten to the first power.

$$\begin{array}{r} x = 0.6\overline{3} \\ \hline 9x = 5.7 \end{array}$$ Subtract the original equation.

Solve for x.

$x = \frac{5.7}{9}$ Eliminate the decimal from the numerator by multiplying both numerator and denominator by ten.

$x = \frac{57}{90}$ Thus, $0.6\overline{3} = \frac{57}{90}$.

The problem can be quickly done mentally, with a minimum of calculations. Using Nick's method, we simply subtract the nonrepeating portion of the decimal (6) from the entire repeating decimal (ignore the bar over the 3): $6\overline{3} - 6 = 57$. This gives the numerator of the fraction. Since there was one digit repeating, use one nine in the denominator. The original decimal had two places, therefore fill in the remaining place in the denominator with a zero. Answer: 57|90.

The general procedure for Nick's method is as follows:

1. Subtract the nonrepeating portion from the entire decimal (ignore the bar).
2. Use one nine in the denominator for each digit that is repeating.
3. Since the denominator must have the same number of decimal places as the original problem, fill in the remaining places with zeros.

Consider the following three problems:

1. Change $0.12\overline{31}$ to a fraction.

Formal method

$$\begin{array}{r} x = 0.12\overline{31} \\ 100x = 12.31\overline{31} \\ \hline x = 0.12\overline{31} \\ \hline 99x = 12.19 \end{array}$$

$x = \frac{12.19}{99}$

$x = \frac{1219}{9900}$

Nick's method

1231 − 12 = 1219. Subtract the nonrepeating portion. Use two nines because there are two digits repeating. The original decimal had four places, so fill in the remaining two places with zeros. Answer: 1219/9900.

2. Change $0.\overline{41}$ to a fraction.

Formal method

$$\begin{aligned} x &= 0.\overline{41} \\ 100x &= 41.\overline{41} \\ x &= 0.\overline{41} \\ \hline 99x &= 41 \end{aligned}$$

$$x = \frac{41}{99}$$

Nick's method

No subtraction is necessary because no portion of the decimal is nonrepeating. Use two nines because there are two digits repeating. No fill-in zeros are necessary because the original decimal only had two places. Answer: 41|99.

3. Change $0.00\overline{3}$ to a fraction

Formal method

$$\begin{aligned} x &= 0.00\overline{3} \\ 10x &= 0.03\overline{3} \\ x &= 0.00\overline{3} \\ \hline 9x &= 0.03 \end{aligned}$$

$$x = \frac{0.03}{9}$$

$$x = \frac{3}{900}$$

Nick's method

No subtraction is necessary. Use one nine because only one digit is repeating. The original decimal had three places, so fill in the remaining two places with zeros. Answer: 3/900.

MINICALCULATORS IN THE CLASSROOM

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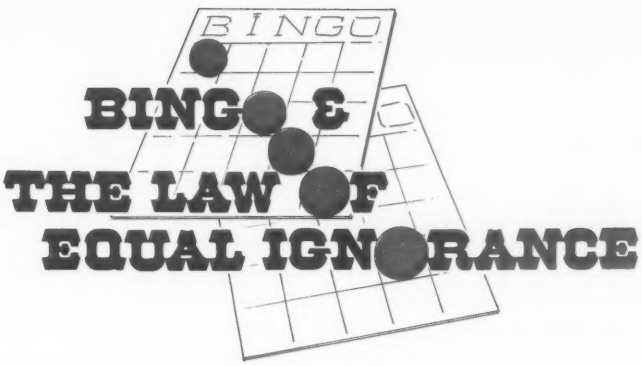
The NCTM favors the appropriate use of minicalculators in the classroom as a valuable instructional aid and encourages the education community to use them in thought-provoking, imaginative ways to reinforce learning and to motivate students as they become proficient in mathematics.

This booklet answers the questions being asked about the use of minicalculators. **How to select a minicalculator? Is it a friend or foe? Just pushing buttons or sound learning?**

The minicalculator booklet is the November 1976 issue of the **ARITHMETIC TEACHER**, an official journal of the National Council of Teachers of Mathematics. Order your copy now and be informed of what the mathematics education community thinks about minicalculators. $2 each.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
1906 Association Drive, Reston, Virginia 22091

(Discounts on quantity orders of the same title shipped to one address are as follows: 2-9 copies, 10%; 10 or more copies, 20%. All orders totaling $20 or less must be accompanied by full payment in U.S. currency or equivalent. Make checks payable to the NCTM. Shipping and handling charges will be added to all billed orders.)



THEODORE LAI

At the time this article was submitted, Theodore Lai was an undergraduate mathematics student at Hunter College of the City University of New York. Currently, he is pursuing graduate studies in mathematics and mathematics education at Hunter. He is also a graduate assistant with the mathematics department at Hunter and a volunteer instructor at the Hunter College Campus School.

Bingo, like other games of chance, can be used in teaching ideas of probability. An introduction to probability may begin with simple experiments using the "law of equal ignorance," which is the basis of the theory of probability. In a *fair* experiment, each possible outcome has the same chances to occur. That is, each outcome is as "ignorant" as the other. In an *unfair* experiment, an outcome is favored to occur over the other possible outcomes; the favored outcome is not as "ignorant" as the other.

In more mathematical terms, if S is the set of all possible outcomes of some given experiment, then by the law of equal ignorance all outcomes in S are equally likely. All outcomes have the same probability of occurrence. In other words, there is no way of knowing which outcome will occur. If S contains n outcomes, then the probability of each outcome is $1/n$. Moreover, if an event A is a subset of S and there are k elements in A, then the probability of A is k/n. That is, the probability that A will occur is the ratio between the number of ways (k) that A can occur and the number of ways (n) that S can occur. Symbolically, we write this as

$$P(A) = k/n.$$

For example, if you toss a coin there are two possible outcomes, heads or tails, and the two outcomes are equally likely. Thus in tossing coins, S = {heads, tails}. Let A be the event that a head will show up; then A = {heads}. Since there is one possibility in A and there are two possibilities in S, $P(A) = 1/2$. A head can occur in one way out of two equally likely ways. The probability of getting tails is the same.

Now let's apply the law of equal ignorance to bingo. The standard 5-by-5 bingo card is arranged as follows:

1. numbers in the first, or "B," column are selected from the numbers 1 through 15;
2. numbers in the next, or "I," column are selected from the numbers 16 through 30;

3. numbers in the next, or "N," column are selected from the numbers 31 through 45;
4. numbers in the next, or "G," column are selected from the numbers 46 through 60; and
5. numbers in the last, or "O," column are selected from the numbers 61 through 75.

Using this information and the law of equal ignorance, we can determine numbers "favored" to win in bingo.

Of the fifteen numbers used in the B column, nine are single-digit numbers and six have two digits. The set of all possible outcomes, S, for the selection of B numbers consists of the numbers from 1 through 15. Let C be the subset of S containing the single-digit numbers and D be the subset whose elements have two digits. By the law of equal ignorance, $P(C) = 9/15$ and $P(D) = 6/15$. Since $P(C)$ is larger, a bingo card with more single-digit numbers in the first column is theoretically better for a win than a card with more two-digit numbers in the same column. Similarly, we can compute probabilities for other types of outcomes in the other four columns.

Notice that ten of the I numbers are in the twenties, four are in the teens, and only one is in the thirties. Therefore, $P(E) = 10/15$, $P(F) = 4/15$, and $P(H) = 1/15$ where $E = \{20, 21, \ldots, 29\}$, $F = \{16, 17, 18, 19\}$ and $H = \{30\}$. The "favored" card should have many spaces covered by numbers in the twenties.

For the next column, we see that there are more numbers in the thirties than in the forties. Therefore, a card is more likely to win with more numbers in the thirties than in the forties in the N column. Symbolically, if $J = \{31, 32, \ldots, 39\}$ and $K = \{41, 41, \ldots, 45\}$, then $P(J) = 9/15$ and $P(K) = 6/15$.

If $L = \{46, 47, 48, 49\}$, $M = \{50, 51, \ldots, 59\}$, and $Q = \{60\}$, then we have $P(L) = 4/15$, $P(M) = 10/15$, and $P(Q) = 1/15$, and $P(M) > P(L) > P(Q)$. Thence the card with more entries in the fifties than in the forties or sixties under G is mathematically favored to win.

Finally, the O numbers are in the sixties and seventies. If R is the subset $\{61, 62, \ldots, 69\}$ and T is the subset $\{70, 71, \ldots, 75\}$ then $P(R) = 9/15$ and $P(T) = 6/15$. Thus a card with more numbers in the sixties than in the seventies under O is more apt to be a winner.

Using the law of equal ignorance, additional discoveries can be made. Children should not find it difficult to apply this aspect of probability to bingo; they may enjoy studying probability with this device. Other simple games of chance, such as those using dice, spinners, or cards, may also be employed to study elementary facts of probability. Provide the students with materials—dice, bingo paraphernalia, spinners, and so on—and let them experiment and test the rules of probability in game situations.

Probability today plays an important role in society. It is evident not only in recreational activities but also in physics, biology, genetics, psychology, economics, political science, sociology, engineering, business, and education. The extent of its applications indicates the remarkable strides that have been made in the theory of probability since its beginning with games of chance in the early seventeenth century.

Build a better geoboard . . .

JUDITH S. KLEIN

As assistant professor of education at Queens College of the City University of New York, Judith Klein teaches elementary science and mathematics methods. In her work with student teachers, primary emphasis is placed on teaching mathematics and science with teacher-made manipulative devices.

In these days when the necessity for economy sometimes pinches good intentions, many teachers are being forced to make do with old instructional materials, to improvise replacements, or to teach without the benefits that manipulative materials bring to the classroom. Although teachers are increasingly aware of the many fine materials that are available to assist in elementary mathematics instruction, many of the commercially produced aids are frequently too expensive to be considered seriously in class quantity. The geoboard is an example.

The typical geoboard consists of a flat piece of wood or plastic that has vertical pegs or nails spaced equidistantly from each other to form a square grid. (Fig. 1) Colored rubber bands can be hooked around the pegs to show geometric relationships involving size, shape, perimeter, area, and so on. Thus, in figure 2, for example, we see how a concave hexagon can be formed on a geoboard.

Fig. 1

My purpose here is not to expound the virtues of the geoboard. Suffice it to say that as an aid to allowing children to learn about things geometric, the geoboard has a multitude of uses. (The references listed at the end of this article include ideas for the actual use of the geoboard in the classroom.) Instead, my purpose is to describe a simple, inexpensive, safe, and perhaps even esthetically pleasing way for teachers to make their own geoboards.

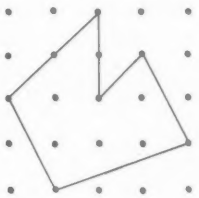


Fig. 2

Many teachers have had the experience of making geoboards from nails and scrap wood with upper elementary students, and have found the experience disagreeable enough that they do not want to repeat it. Not only were they concerned about

mashed thumbs, but they also found that the quality of workmanship left a great deal to be desired. Nails got hammered in at strange angles. Even when the work was spread over several short sessions, children quickly tired of hammering. Many of the homemade geoboards do not stand up well in use. Nails bend or fall out; and even in the far from tropical dampness of New York City, nails often rust and, if nothing else, look rather unsightly. Finally, the nail heads, which conveniently prevent rubber bands from flying, also present quite a menace to small hands that approach the board from the wrong end.

My solution to many of the above problems consists of the substitution of push-pins (fig. 3), sometimes known as "draftsman's tacks" for the nails. The pin portions are fairly sharp and thus are easily

Fig. 3

inserted into a board made of soft wood. The plastic tops restrain the free flight of rubber bands much as the nail heads do, without the attending problems. Push-pins also are available in a variety of colors, from clear to bright red and blue. Thus students may select color combinations that appeal to them, thereby putting their own distinctive touches on the geoboard. Economy is well served, too; push-pins can be had for well under a penny apiece when bought in large quantities (by the thousand or even by the gross) and scrap wood from a lumber yard costs nothing. Moreover, by making their own geoboards, teachers can specify their own grid dimensions. Most commercial models feature a 5-by-5 or 6-by-6 grid. It is not difficult to understand that a 10-by-10 grid, for example, could extend the uses of the geoboard, as well as allow students to generate a more varied set of data for each problem to be solved.

Finally, a few technical notes on construction. Soft wood such as pine is probably best. It can be finished easily by sanding with a medium grade sandpaper, and then waxing. Alternatively, some students may prefer to cover their geoboards with solid-colored, self-stick vinyl. So far as providing a uniform grid, the most convenient way to do this is to prepare the desired grid system of dots on a stencil and run off at least one copy per student. Dots can be spaced two centimeters apart. The resulting dot paper can then be superimposed on each board, and a pin stuck in at each dot—once all the pins are in place, the dot paper can be removed. Pins can be fixed permanently in place by placing a small drop of glue under each head, but this is certainly not a necessity. Once the pins are in place, the geoboard is ready for use. It is a cheap, safe, and durable piece of apparatus, and, above all, it will give students a feeling of personal investment in many of their mathematical experiences.

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Your Council at Work

THE EDITORIAL PANEL OF THE *MATHEMATICS TEACHER*

The *Mathematics Teacher*'s Editorial Panel is responsible for creating the overall policy regarding the reviewing of manuscripts. It is also responsible for balancing the journal's content and for creating special sections. Their goal is to produce a journal that serves the interests of mathematics teachers in grades seven through fourteen.

Each manuscript, before being reviewed by the Panel, is considered by at least three referees who are selected so as to include appropriate combinations of high school teachers, teacher educators, and mathematicians. (Nearly five hundred referees were acknowledged in the December 1976 issue of the *Mathematics Teacher*, and they reviewed over six hundred manuscripts during 1976.) The Editorial Panel evaluates each manuscript approved by the referees and makes the final decision concerning its acceptance or rejection.

Members of the Editorial Panel are appointed by the Council's president for a term of three years. A member of the Board of Directors is also appointed to the Panel as a liaison member, serving for the duration of his or her term with the Board. The members of the Panel (excluding the nonvoting managing editor) currently include two classroom teachers, two supervisors, and two teacher educators, representing an average of over twenty years of classroom teaching experience.

The managing editor of the *Mathematics Teacher* corresponds with authors, assigns manuscripts to reviewers, schedules manuscripts for publication, and otherwise carries out the policies of the Panel. He is responsible for the editing of manuscripts and for overseeing the preparation of artwork for the journal.

The Editorial Panel meets twice a year to consider proposals relating to the content and policy of the journal. Changes in journal policy or procedures that involve budgeting allocations are considered by the Board of Directors. This past year, the Panel approved a series of Bicentennial articles, included a special issue on individualization, and attempted to give greater vitality to the "Reader Reactions" section of the journal. It is currently working on establishing a regular section on applications.

Readers' ideas for the journal are always welcome.

Eugene Nichols, *Chairman*

Left to right: James T. Fey; Thelma M. Sparks; Gladys M. Thomason; Eugene D. Nichols, *Chairman;* Harry B. Tunis; and Lawrence D. Hawkinson. *Not shown:* Richard A. Hanson.

ANNUAL FINANCIAL REPORT

The National Council of Teachers of Mathematics further strengthened its sound financial position during the 1975–76 fiscal year. In spite of a projected deficit, a soaring rate of inflation, and necssary expenditures beyond those budgeted, NCTM ended the year with receipts over expenditures totaling $27 511.

The financial planning for 1976–77 took into account the fact that requests for funds exceeded projected revenue by a significant amount. A dues and subscriptions increase had to be proposed in order to maintain a balanced budget. NCTM is in an enviable position among professional organizations from a financial point of view, thanks to the efforts of many responsible leaders. Our total net worth has risen dramatically with the completion of the headquarters building, and we also have a sizable special reserve fund.

James R. Smart, *Chairman*
David W. Wells
Helen Kriegsman
John Devlin
Joseph Stipanowich
James D. Gates
James R. Tewell (nonvoting)

GENERAL FUND

	Summary Budget Fiscal Year 1 June 1975 to 31 May 1976	Receipts and Expenditures Fiscal Year 1 June 1975 to 31 May 1976	Summary Budget Fiscal Year 1 June 1976 to 31 May 1977
RECEIPTS			
Memberships	$ 555 000	$ 543 804	$ 602 800
Institutional Subscriptions	390 000	396 268	408 700
Subscriptions to the *Mathematics Student*	18 000	19 028	25 000
Subscriptions to the *Journal for Research in Mathematics Education*	31 000	32 562	38 000
Back Issues of Journals	3 000	6 421	2 600
Reimbursement of Shipping Expenses	25 000	15 351	13 800
Journal Advertising	113 570	119 473	112 000
Affiliated Group Dues	2 500	2 801	2 500
Rental of Mailing Lists	15 000	14 511	12 000
Publication Sales	273 600	279 047	334 786
Convention Registration Fees	126 270	88 391	114 000
Convention Exhibit Fees	134 700	163 221	140 000
Convention Program Advertising	18 480	23 315	20 000
Other Convention Income		4 237	4 200
Miscellaneous	500	17 208	900
Total Receipts	$1 706 620	$1 725 638	$1 831 286
EXPENDITURES			
Administrative Expenses	$ 801 550	$ 789 643	$ 882 265
Building Mortgage/Operation	123 300	123 300	126 400
Subscription Agency Commissions	36 700	34 479	36 100
Journals	328 580	324 665	321 905
Newsletter	9 000	11 390	11 000
Officers and Board	104 650	91 899	78 910
Committees	102 979	78 687	109 571
Publications	87 600	82 286*	117 058
Convention Programs	45 500	52 325	49 000
Convention Speakers	19 000	11 016	19 000
Other Convention Expenses	69 300	74 492	74 200
Election	10 000	8 537	10 000

*Additional publications with a production cost of $35 281.01 were printed with a special allocation of funds from the 1974–75 surplus.

Miscellaneous		15 408	
Special Reserve Fund	(31 539)	(31 539)	(4 123)
Total Expenditures	$1 706 620	$1 666 588	$1 831 286

AUDITOR'S REPORT

We have examined the accompanying Balance Sheet of Funds of The National Council of Teachers of Mathematics as of 31 May 1976 and the related Statements of Revenues, Expenses, and Fund Balances prepared for the fiscal year then ended. The Council, with Board approval and on a progressive basis, commenced the conversion of its accounting system from the cash to the accrual basis effective for the fiscal year ended 31 May 1976. Accordingly, except for the recognition of Memberships, Subscriptions, and Accrued Annual Leave, which are planned for accrual basis recognition in the fiscal year commencing 1 June 1977, the statements examined and submitted were prepared on this basis. Our examination was made in accordance with generally accepted auditing standards and accordingly included such tests of the accounting records and such other auditing procedures as we considered necessary in the circumstances.

Subject to the foregoing comments, in our opinion, the accompanying Balance Sheet of Funds and Related Statements of Revenues, Expenses, and Fund Balances fairly presents, in conformity with generally accepted principles of accounting, the financial position of The National Council of Teachers of Mathematics as of 31 May 1976 and the results of its activities for the fiscal year then ended.

Raymond H. McGivern
Certified Public Accountant

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
BALANCE SHEET OF FUNDS
MAY 31, 1976

ASSETS

GENERAL FUND			
Cash		$ 100 299.88	
Accounts Receivable		65 964.34	
Inventory of Publications		279 364.38	
Prepaid Expenses		19 833.31	
Due from Other Funds:			
Life Membership Fund	$ 122.30		
Building Fund	16.13	138.43	
			$ 475 600.34
SPECIAL RESERVE FUND			
Cash		$ 155 191.34	
Investments at Cost—Market Value $249 820.75		225 893.61	
Due from Other Funds:			
General Fund	$ 79 806.45		
Building Fund	40 000.00	119 806.45	500 891.40
BUILDING AND RENTAL FUND			
Cash		$ 29 608.63	
Guarantee Deposit		1 500.00	
Fixed Assets:			
Land and Building at Cost	$ 895 107.11		
Furniture and Equipment at Cost	122 471.55		
Total	$1 017 578.66		
Less Accumulated Depreciation	94 380.28	923 198.38	954 307.01

LIFE MEMBERSHIP FUND			
Cash		$ 44 348.84	
Investments at Cost—Market Value $42 910.63		39 987.82	84 336.66
FRENCH TRANSLATION FUND			
Accounts Receivable:			
Mathematique de Quebec			2 479.47
NAEP GRANT FUND			
Due from General Fund			1 006.34
			$2 018 621.22

LIABILITIES AND FUND BALANCES

GENERAL FUND			
Liabilities:			
Accounts Payable and Accrued Salaries	$ 42 571.98		
Deposit—Tenants and Others	5 213.00		
Payroll Withholdings	1 627.16		
Other	4 342.06		
Due to Other Funds:			
Special Reserve Fund $79 806.45			
Grant Fund 1 006.34	80 812.79	$ 134 566.99	
Fund Balances:			
Restricted—NSF $(6 267.42)			
NOVUS 2 311.36	$ (3 956.05)		
Nonrestricted	344 989.40	341 033.35	$ 475 600.34
SPECIAL RESERVE FUND			
Fund Balance			500 891.40
BUILDING AND RENTAL FUND			
Liabilities:			
First Deed of Trust Mortgage Note	$ 462 805.66		
Accounts Payable and Accrued Salaries	806.41		
Due to Other Funds:			
Special Reserve Fund $40 000.00			
General Fund 16.13	40 016.13	$ 503 628.20	
Fund Balance		450 678.81	954 307.01
LIFE MEMBERSHIP FUND			
Liabilities:			
Due to General Fund and Other		$ 123.08	
Fund Balance		84 213.58	84 336.66
FRENCH TRANSLATION FUND			
Fund Balance			2 479.47
NAEP GRANT FUND			
Fund Balance			1 006.34
TOTAL			$ 2 018 621.22

COMMITTEES AND REPRESENTATIVES, 1976–77 SUPPLEMENTAL LIST

This list supplements the report "Officers, Directors, Committees, Projects, Representatives, and Executive Staff (1976–77)," which appeared in the October 1976 issues of the *Arithmetic Teacher* and the *Mathematics Teacher.*

Mathematics Education Trust Committee

Helen F. Kriegsman, Pittsburg, Kansas, Chairman; 1978

Donovan A. Johnson, Minneapolis, Minnesota; 1979

E. Glenadine Gibb, Austin, Texas; 1980
Leroy Sachs, Clayton, Missouri; 1981
Geraldine Green, Royal Oak, Michigan; 1982

Research Advisory Committee
Terrence Coburn, Pontiac, Michigan; 1978 (replacing Betty J. Hight, who resigned)

External Affairs Committee
Second NCTM Representative to High School Mathematics Contest Committee
Thomas J. Englert, Cleveland Heights, Ohio; 1980

Committee to Review "Infinity Factory"
Catherine Tobin, Newtonville, Massachusetts; Chairman
Patricia Davidson, Boston, Massachusetts
Mary Ann Jackson, Roxbury, Massachusetts
Hattie McKinnis, Dorchester, Massachusetts
Jean McManamy, Cambridge, Massachusetts

Publications Committee
Professional Reference Book
Mathematical Education of Exceptional Children and Youth
Vincent J. Glennon, Storrs, Connecticut, Editor

Meetings Committee
Missoula Meeting, 16–18 March 1978, Missoula, Montana
Jim Burrington, Missoula, Montana, Convention Cochairman
Rick Billstein, Missoula, Montana, Convention Cochairman
Lee Von Kuster, Missoula, Montana, Program Chairman

Your Professional Dates

NCTM 55th ANNUAL
20–23 April 1977 Cincinnati, Ohio

NCTM 56th ANNUAL
12–15 April 1978 San Diego, California

NCTM 57th ANNUAL
18–21 April 1979 Boston, Massachusetts

WESTERN MEETINGS

January

29: *Santa Clara Valley Mathematics Association,* Santa Clara, Calif. (Jean J. Pedersen, Dept. of Mathematics, University of Santa Clara, Santa Clara, CA 95053)

February

5: *Orange County Mathematics Association,* Costa Mesa, Calif. (Dennis Titzkowski, Santiago High School, 12342 Trask Ave., Garden Grove, CA 92643)

24–26: NCTM PHOENIX MEETING, Arizona

25: *National Council of Supervisors of Mathematics,* Phoenix, Ariz. (Ron McCully, Phoenix Union High School, 2526 W. Osborn Rd., Phoenix, AZ 85017)

SOUTHWESTERN MEETINGS

January

27–28: *Kansas Association of Teachers of Mathematics,* Wichita, Kans. (Forrest L. Coltharp, Kansas State College of Pittsburg, Pittsburg, KS 66762)

February

1: *Alamo District Council of Teachers of Mathematics,* San Antonio, Tex. (Sr. M. Scholastica, Blessed Sacrament Academy, 1135 Mission Rd., San Antonio, TX 78210)

8: *Houston Council of Teachers of Mathematics,* Houston, Tex. (Catherine Waldmann, Madison Senior High School, 6249 Longmont, Houston, TX 77027)

17–19: NCTM LITTLE ROCK MEETING, Arkansas

18: *National Council of Supervisors of Mathematics,* Little Rock, Ark. (Nancy Biggs, Memphis City Schools, 2225 James Rd., Memphis, TN 38127)

24: *Bartlesville Council of Teachers of Mathematics,* Bartlesville, Okla. (Janet R. Maloney, College High School, 1800 Hillcrest Dr., Bartlesville, OK 74003)

25–28: *Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics,* New Orleans, La. (Lawrence H. Davis, Box 326 Univ. Sta., Hammond, LA 70401)

NORTH CENTRAL MEETINGS

February

5: *Minnesota Council of Teachers of Mathematics,* Bloomington, Minn. (Elmer Koch, Jr., 9218 10th Ave. South, Bloomington, MN 55410)

CENTRAL MEETINGS

January

11: *Greater Toledo Council of Teachers of Mathematics,* Toledo, Ohio (George Shirk, University of Toledo, West Bancroft St., Toledo, OH 43606)

13: *Gary Area Council of Teachers of Mathematics,* Gary, Ind. (Elizabeth Martin, St. Mary's School, 405 E. Joliet Street, Crown Point, IN 46307)

14: *Metropolitan Mathematics Club of Chicago,* Chicago, Ill. (Jerry Cummins, Proviso West High School, Wolf & Harrison, Hillside, IL 60162)

20: *Elementary School Mathematics Advisors of the Chicago Area,* Willowbrook, Ill. (Jack Hotsenpiller, District U-46, 4 S. Gifford St., Elgin, IL 60120)

27: *Cleveland Diocesan Mathematics Teachers Association,* Cleveland, Ohio (Francis L. Macuga, 333 E. 210th, Cleveland, OH 44119)

February

5: *Northeast Ohio Tri-County Council of Teachers of Mathematics,* Willoughby, Ohio (Van L. Speers, Willowick Junior High, 31500 Royalview Dr., Willowick, OH 44094)

9: *Gary Area Council of Teachers of Mathematics,* Gary, Ind. (Elizabeth Martin, St. Mary's School, 405 E. Joliet St., Crown Point, IN 46307)

17: *Metropolitan Mathematics Club of Chicago,* Westchester, Ill. (Jerry Cummins, Proviso West High School, Wolf & Harrison, Hillside, IL 60162)

SOUTHEASTERN MEETINGS

February

2: *Tidewater Council of Teachers of Mathematics,* Suffolk, Va. (Blanche Lipman, Western Branch High School, 4222 Terry Drive, Chesapeake, VA 23321)

17: *Peninsula Council of Mathematics of Virginia,* Hampton, Va. (Charlotte E. Remaley, Y. H. Thomas Annex, 1300 Thomas St., Hampton, VA 23669)

NORTHEASTERN MEETINGS

January

29: *Association of Mathematics Teachers of New Jersey,* Cherry Hill, N.J. (Ken Solem, Gloucester College, Tanyard Road, Sewell, NJ 08080)

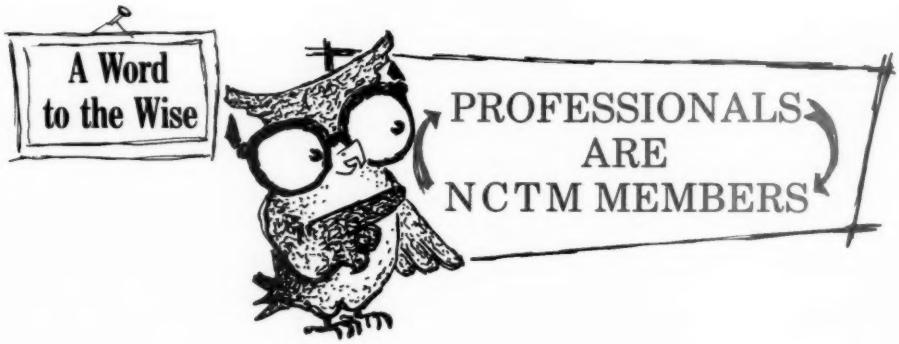
February

26: *Association of Teachers of Mathematics of Philadelphia and Vicinity,* Philadelphia, Pa. (Mark Wiener, West Chester State College, Dept. of Mathematical Sciences, West Chester, PA 19380)

OTHER MEETINGS

January

29–31: *Mathematical Association of America,* St. Louis, Mo. (A.B. Willcox, Mathematical Association of America, 1225 Connecticut Ave., NW, Washington, DC 20036)



NATIONAL COUNCIL OF

Teachers of Mathematics

Publisher of the *Mathematics Teacher* and the *Arithmetic Teacher*

Also publishes a variety of major references on mathematical topics and pedagogical methods concerning mathematics. Here are some of the most recent (all are clothbound):

***Mathematics Learning in Early Childhood*, 37th Yearbook.** Colorful, abundantly illustrated resource book for teaching mathematics to children aged 3-8. Chapters on cognition, curriculum, research, and teaching procedures are highlighted by hundreds of ideas and activities in an 8½-by-11-inch format. 1975, 316 pp., $13.00*

***Geometry in the Mathematics Curriculum*, 36th Yearbook.** Presents the various theories on how geometry might best be taught at all levels—informally from kindergarten through the two-year college as well as formally at the secondary level, with illustrations given for each formal approach (conventional, coordinate, transformation, affine, vector). 1973, 480 pp., $11.50*

***The Slow Learner in Mathematics*, 35th Yearbook.** Provides ideas for teaching the slow learner at all levels and deals with subject matter objectives while emphasizing methods for attaining them. 1972, 528 pp., $12.20*

***Instructional Aids in Mathematics*, 34th Yearbook.** A richly illustrated guide to instructional aids, a basis for evaluating their quality and utility, suggestions for their use, and ideas for their construction, all in a larger format (9-by-11 inches). 1973, 442 pp., $14.60*

***The Teaching of Secondary School Mathematics*, 33d Yearbook.** Forces shaping today's mathematics program are described; teaching for special outcomes is discussed; then examples demonstrate classroom applications, with emphasis on teacher planning. 1970, 433 pp., $10.90*

***A History of Mathematics Education in the United States and Canada*, 32d Yearbook.** Issues and forces affecting grades K-12 from colonial days to the present. 1970, 557 pp., $12.20*

***Historical Topics for the Mathematics Classroom*, 31st Yearbook.** A substantial treatment of the use of the history of mathematics in the teaching of mathematics. 1969, 524 pp., $11.95*

*NCTM members are entitled to a $2 discount (one copy only) on each yearbook.

All orders totaling $20 or less must be accompanied by payment in U. S. currency or equivalent. Make checks payable to the National Council of Teachers of Mathematics. Shipping and handling charges will be added to all billed orders.

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An annotated listing of all NCTM publications is free on request.

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Diagnostic and Prescriptive Techniques in Mathematics

March 31—April 2, 1977 College Park, Maryland

The Arithmetic Center of the University of Maryland and the Maryland State Department of Education are sponsoring a conference on diagnostic and prescriptive mathematics. 20 national leaders in elementary mathematics will be presenting talks and workshops for teachers and specialists. Theoretical presentations for researchers will be featured in a one-day post-conference.

Help from national leaders for your slow students in mathematics

For complete information, clip and mail to:

Dr. J. Dan Knifong, Conference Coordinator
Arithmetic Center
University of Maryland
College Park, Maryland 20742

Or call (301) 454-4941

Name _____

Affiliation _____

Address _____

City _____ **State** _____ **Zip** _____

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